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2021
JUNE

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Interest Rate Rules, Rigidities and Inflation Risks in a Macro-Finance Model

(Kamatszabályok és ármerevség hatása az inflációs kockázatokra egy makro-pénzügyes modellben)

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Budapest, June 2021

Published by the Magyar Nemzeti Bank

Publisher in charge: Eszter Hergár

Szabadság tér 9., H-1054 Budapest

www.mnb.hu

ISSN 1585-5600 (online)

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****We benefited from discussions with Gianluca Benigno, Alessia Campolmi, Max Gillman, Michal Kejak, Istvan Konya, Patrick Minford, Panayiotis Pourpourides, Eric Swanson and Mike Wickens. We are grateful for the comments received at the CEF, EEA, MMF, Dynare and EBES conferences. Horvath acknowledges the support of the Czech Science Foundation no. 20-17044S and Charles University Research Centre program No. UNCE/HUM/035. Contacts: roman.horvath@fsv.cuni.cz, kaszabl@mnb.hu, ales.marsal@nbs.sk

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Abstract

Long-term bond yields contain a risk-premium, an important part of which is compensation for inflation risks. The substantial increase in the Fed funds rate in the mid-2000s did not raise long-term US Treasury yields due to the reduction in the term premium (so-called Greenspan conundrum) which was typically thought to be exogenous for monetary policy. We show using a New Keynesian macro-finance model that the term premium is endogenous and is greatly influenced by the specification of the Taylor rule. Finally, we extend the model with frictions (richer fiscal setup and wage rigidity) that are known to help jointly match macro and finance data and estimate the model on US data in 1961-2007 by the generalized methods of moments and simulated methods of moments.

JEL: E13, E31, E43, E44.

Keywords: zero-coupon bond, nominal term premium, inflation risk, Taylor rule, New Keynesian, labor income taxation, wage rigidity, GMM, SMM.

Összefoglaló

A hosszabb lejáratú (pl. 10 éves) államkötvények hozama tartalmaz kockázati prémiumot, amely elsősorban a jövőbeni magasabb infláció esetén kompenzálja a kötvény tulajdonosát (nem-indexált államkötvények esetén). A 2000-es évek közepén az USA központi bankjának szerepét betöltő Fed többszörös kamatemelésre kényszerült, azonban a hosszabb lejáratú eszközök kamatai - a szokásos logika (hozamgörbe várakozási hipotézise) szerint - nemhogy emelkedtek, hanem inkább csökkentek (az akkori jegybank elnök után ezt Greenspan rejtélyként tárgyalja a szakirodalom), mivel a hosszú lejáratú kötvények hozamában a kockázati prémium jelentősen csökkent. Ekkoriban még széles körben elterjedt volt az a nézet, hogy a kockázati prémium exogén módon változik a monetáris politikától függetlenül. Egy új-keynesi makro-pénzügyes modellben megmutatjuk, hogy a hozam kockázati prémium endogén és nagyban befolyásolja a monetáris reakció függvény vagyis a Taylor-féle kamatszabály. A tanulmány utolsó részében olyan frikciókkal egészítjük ki a modellt (részletesebb fiskális szektorral és reál bérragadóssággal), amelyekről ismert, hogy segítik a modell makró és pénzügyi adatokhoz való illeszkedését. Végül a modellt megbecsüljük USA adatokon (1961-2007) általánosított és szimulált momentumok módszerével (GMM és SMM).

1 Introduction

The inflation risk premium contained in the yields of long-term nominal government bonds (Treasuries in the US or gilts in the UK) can be defined as the difference between the nominal and the real term premium, which serves as compensation for the nominal and real risks that the bond holder has to bear. In other words, bond holders expect long-term bonds to pay an additional return, called the term premium, to compensate for future uncertainty over consumption and inflation. Hördahl and Tristani (2012) estimate this premium and provide an overview of the papers seeking to measure inflation risks. These papers estimate the inflation risk premium to be in the range of 10 to 70 basis points, depending on the time period considered and the countries included in the sample.[1]

The key contribution of our paper is the description of how term premium, which is endogenous in our model, is affected by the conduct of monetary policy. In 2004, the Fed embarked on a tightening cycle raising short-term interest rate gradually from one to 5.25 per cent. Still the the long-term US Treasury yields declined seemingly counter-intuitively and Greenspan, the Chairman of the Fed of the time, coined this phenomenon as conundrum. In fact, the tightening cycle led to a shrinkage of the nominal term premium (and its inflation risk component) in the yields of long-term bonds. Previous research treated the nominal term premium exogenous from the point of view of the conduct of monetary policy. Our paper is the first to show that monetary policy described by the specification of the Taylor rule (in normal times when the interest rate has not reached its lower bound) affects term premium to a great extent.[2]

This paper uses a simple New Keynesian macro-finance model with Epstein-Zin recursive preferences to generate inflation risks calculated as the difference between the nominal and real term premium of long-term, default-free, zero-coupon bonds. Our model captures the magnitude of the inflation risk premium in the data.[3] The model in this paper employs the framework of Rudebusch and Swanson (2012) (henceforth, RS), who use Epstein-Zin preferences to make households sufficiently risk averse without decreasing the intertemporal elasticity of substitution to counterfactually low levels. In RS and our paper, nominal bonds are risky due to temporary technology shocks that create negative covariance between consumption and inflation. A negative productivity shock, leads, for instance, to a rise in inflation, which erodes real returns on bonds during periods of low consumption and makes nominal bonds a poor hedge.

We contribute to the literature by showing that the specification of the interest rate rule (the so-called Taylor rule) affects inflation risks to a large extent. In particular, (i) the timing assumption on inflation in the interest rate rule and (ii) the inclusion of interest-rate smoothing are important with respect to inflation risks. (iii) The definition of the output gap and the size of the coefficient on the output gap also have sizable impacts on the magnitude of inflation risk premia.[4] (iv) We find that a time-varying inflation target with 'learning'[5] of the inflation target does not substantially increase the mean of the nominal term premium but raises the standard deviations of nominal variables such as inflation. (v) we show that the introduction of price rigidity elevates inflation risks with Epstein-Zin preferences in contrast to previous papers which argue for the opposite effect with consumption habits. As a last point, (vi) we estimate an extended version of the model by GMM and SMM on quarterly US data over 1961-2007.

First, we consider (i). It can be shown that the less focused the central bank is on current inflation, for example, because it seeks to stabilize inflation in the medium run,[6] the more weight is given to real economy considerations (captured by the output gap

[1] For instance, Buraschi and Jiltsov (2005) find that the 10-year US treasury features a risk-premium of 70 basis points (bps) over 1960-2002. Ang et al. (2008) estimate 115 bps for the 5-year US treasury over 1952-2004. Papers which include information on indexed (real) bonds typically find lower estimates (see, e.g., D'Amico et al. (2010) who obtain an estimate of 50 bps on average for a 10-year US nominal bond over 1990-2006).

[2] Importantly, we do not claim that our paper is the first where the term premium is endogenous. In this paper we describe how monetary policy affects term premium, with a special focus on inflation risk premium, in a model where the term premium is endogenous.

[3] In section eight of this paper we estimate an extended version of the model that is able to match nominal term premium as well.

[4] Unlike estimates of the output gap coefficient, there is consensus in the literature on the size of the coefficient of inflation in the Taylor rule. A higher coefficient on inflation, *ceteris paribus*, generally reduces inflation risks.

[5] Our model does not explicitly model learning but, instead considers a shortcut to learning whereby the inflation target is updated in each period relative to a moving average of current and past inflation (an adaptive rule).

in the Taylor rule) and, thus, the lower is the real term premium on long-term bonds. Importantly, the inclusion of time $t+1$ inflation in the Taylor rule implies that inflation at time t is not stabilized, thereby elevating inflation risks and the yields of long-term bonds through a jump in nominal term premia.

Alternatively, the inclusion of current-period or future inflation in the Taylor rule can be replaced by a moving average of current and past inflation rates, which delivers an average inflation rate that is less affected by current shocks and, therefore, calls for a smaller reaction of the nominal interest rate. In particular, the inclusion of an average of current and past inflation rates in the Taylor rule, as in RS and in several medium-sized DSGE models such as the Smets and Wouters (2007) model, reduces the influence of time-t shocks on the measure of inflation in the interest rate rule and leads to a reduction in consumption and inflation risks.

Next, we consider (ii). Several papers find that some type of inertia in the nominal interest rate is necessary to model the US monetary policy (see, e.g., Rudebusch (2006) and Carillo et al. (2007) for recent attempts).[7] Here, we discuss the macro-finance implications of introducing monetary policy inertia by lagging the nominal interest rate. In a response to movements in inflation and the output gap, the nominal interest rate will rise less due to interest rate smoothing. We note that a smaller rise in the nominal interest implies less movement in the real interest rate, and therefore, consumption, hours worked and output will also be less restrained. By the same token, the labor supply channel that households use to insure against bad shocks is more powerful when there is interest rate smoothing. As a result, the inflation risks will be lower.

We now turn to elaborate on (iii). Inflation risks are also influenced by the size of the coefficients on inflation and the output gap in the Taylor rule. Empirical estimates of the output gap coefficient are typically close to either zero or one (see Table 1, which is based on the estimates by Clarida et al. (1998, 2000) on US data). The higher the coefficient on the output gap is, *ceteris paribus*, the lower are real risks and the more elevated are inflation risks, as the central bank cares more about stabilizing the output gap relative to mitigating deviations of inflation from its target. An output gap coefficient that is close to one ensures that the real risks are virtually zero, as in the context of the RS model. A higher coefficient on inflation in the Taylor rule, however, indicates that policy makers are more interested in stabilizing fluctuations in inflation, indicating a reduction in inflation risks. Recent estimates of the coefficient on the growth rate of the output gap for the period of the Great Moderation in the US (see, e.g., Coibion and Gorodnichenko (2011)) are even higher than one.

Table 1
Taylor rule estimates of Clarida et al. (1998, 2000) for the US

	ρ	ϕ_π	ϕ_y
Rule 1 (Clarida et al. 1998) for 1979-1994	0.92	1.79	0.07
Rule 2 (Clarida et al. 2000) for 1983-1996*	0.79	2.16	0.93

Notes: This table is borrowed from Kaszab and Marsal (2015). Clarida et al. (1998, 2000) estimate the following forward-looking Taylor rule: $i_t = \rho i_{t-1} + (1-\rho)[\phi_\pi \bar{\pi}_{t+1} + \phi_y y_t]$. *In RS,* $\bar{\pi}_t$ *is used instead of* $\bar{\pi}_{t+1}$*, although we obtained similar results for the case of* $\bar{\pi}_{t+1}$*. *Quite close to the values of RS, who use the estimates of Rudebusch (2002):* $\rho = 0.73$, $\phi_\pi = 2.1$ *and* $\phi_y = 0.93$ *[Remark: in RS, inflation is annualized in their Taylor rule, and therefore,* $\phi_\pi = 0.53$ *is used].*

We now turn to the discussion of point (iv), namely, the time-varying inflation target with 'learning'.[8] It is intuitively appealing that a time-varying inflation target creates uncertainty about the inflation and might lead to higher inflation risks (see, e.g., RS).[9] However, we find that a time-varying inflation target with learning of the inflation target is helpful in amplifying the standard deviations of finance moments but has a negligible effect on the mean of the nominal term premia, for instance.

[6] The expected inflation is included in the Taylor rules of some medium-size DSGE models such as the GIMF of the IMF, see Carabenciov et al. (2008), or the ToTEM model of the Bank of Canada, see Murchison and Rennison (2006).

[7] These papers show that the inertia in monetary policy can be traced back to the persistence in the monetary policy shock rather than autocorrelation in the nominal interest rate. In this paper, we consider the more traditional approach, which is the introduction of a lagged term in the nominal interest rate.

[8] Please note that our model contains only a short-cut of model with learning. In particular, we assume that an adaptive rule is in operation: the actual inflation target is updated by the difference between the actual inflation target and a moving average of past inflation.

As an extension of our baseline model, we use a Taylor rule in which the time-varying inflation target is revised upward (downward) if current inflation is higher (lower) than the current inflation target (similar to the specification in RS). Hence, agents learn about the inflation target in an adaptive way. However, using various specifications of the Taylor rule, we find that a long-run, time-varying inflation target with learning contributes little to inflation risks. Our finding is also consistent with Hördahl et al. (2008), who use of a model with a time-varying inflation target but without learning of the inflation target and with habits in consumption but non-recursive preferences. Hördahl et al. (2008) emphasize that it is not surprising that the inflation risks are zero with the time-varying inflation target as the model that they and we use assume "*perfect credibility of monetary policy and where the inflation target, albeit time-varying, is well-anchored in the long-run and perfectly understood and known by all agents.*" (Hördahl et al. (2008) pp. 1961.)

Next, regarding (v) we contribute by observing that the introduction of price rigidity and/or higher price rigidity in the form of some real rigidity such as firm-specific fixed capital help to increase the nominal term premium.[10] Real frictions such as firm-specific fixed capital have effects that are observationally equivalent to higher price stickiness—they reduce the slope of the New Keynesian Phillips curve. There are at least two reasons why the introduction of real rigidity due to firm-specific input contributes to higher nominal term premium.

First, we argue that the introduction of real rigidity together with Epstein-Zin preferences implies more persistent effects of negative shocks even in the absence of permanent shocks and induces the household to demand higher risk-premium for holding risky long-run bonds. Second, higher real rigidity acts similarly to higher nominal rigidity and raises the standard deviation of price dispersion which reduces the amount of output that can be produced with labor for given amount of capital. Further, higher price dispersion also diminishes the effectiveness of labor which can be used to insure against bad shocks and, hence, households command higher risk-premium.

The introduction of price rigidity increases the persistence of real and nominal variables in the economy. A longer average duration of price stickiness or higher real rigidity induces higher distortions in the economy (in the form of higher price dispersion), and therefore, the labor supply channel that serves as a vehicle to insure against negative shocks has a reduced effect (due to the price dispersion one unit of labor produces less than one unit of output). The negative correlation between consumption and inflation that is required for nominal bonds to be risky is stronger with real or nominal rigidities.

As a further exercise, (vi), we have done normative (welfare) analysis of the Taylor rules used in our paper. The welfare cost is reported in terms of consumption equivalents relative to the deterministic steady state in line with Schmitt-Grohe and Uribe (2007). The simple gap model produces larger output gap and larger welfare costs relative to the flexible price output gap model. The welfare cost is especially substantial in the case of a high coefficient on the output gap.

Finally, (vii) we estimate an extended version of the RS model with either simple or flexible-price output gap on US data over the period 1961-2007 by GMM and SMM. We find that the estimated model, which contains labor income taxation and wage rigidity, matches a set of macro and finance moments quite well. In addition, the model, which contains the simple version of the output gap, has a better fit compared to the version with flexible price output gap. The macroeconomic literature has long used linearised models for estimation (see Le et al. (2016)). However, note that our models aim to capture risk-premia in the yields of risky assets so higher-order approximation (typically third-order) is necessary.

The paper is structured as follows. Section 2 provides a brief literature survey. Section 3 describes the model and the pricing of real and nominal bonds. Section 4 reports on the calibration and solution method. Section 5 presents results of the model with and without a time-varying inflation target. Section 6 contains the welfare analysis. Section 7 focuses on the effect of real and nominal rigidity on nominal and real term premia and inflation risks. Section 8 presents the GMM and SMM estimation of the extended version of the model. The last section concludes.

[9] Note that Doh (2011) finds estimating a New Keynesian model on US data using Bayesian methods that inflation target shocks are the main driver of the yield curve when prices are flexible. Kliem and Meyer-Gohde (2017) use a medium-scale DSGE model but find inflation-target shocks to be important in contributing towards the real term premium component of the nominal term premium.

[10] Note that these frictions are included in the model of RS (2008, 2012), but they do not observe that price stickiness and real rigidities help to raise the nominal term premium. They also do not explain why nominal and real rigidities can better account for the empirically observed size of the nominal term premia.

2 Literature review

Our research is related to several papers in the macro-finance literature. Our paper aligns with RS, who find using the same model as ours that the nominal term premium can be large and volatile. However, RS do not decompose the nominal term premium into the real term premium and inflation risk premium, which we accomplish in this work. Our paper is also related to Hsu et al. (2020), who also obtain a positive inflation risk premium by employing a more detailed model that also includes wage rigidity and process innovation. Kliem and Meyer-Gohde (2017) use a medium-scale New Keynesian model with various temporary and permanent shocks and find that the nominal term premium is mainly a compensation for real risks rather than inflation risks. In our paper, we intentionally use a simple model with only temporary shocks (with productivity shock being the dominant one) so that we can provide a clear picture of how the various types of interest rate rules affect the nominal term premium.

Previous papers (see, e.g., de Paoli et al. (2010)) operate with habit formation whereby the highest market price of risk[11] is attributed to short-run fluctuations (business cycle or shorter length). Whereas, in our paper, the representative household with Epstein-Zin preferences is the most averse to shocks which have permanent and persistent long-run effects (longer than business cycles).

In a related paper, Horvath et al. (forthcoming) suggest that the introduction of a richer fiscal setup (the government budget is balanced by income tax revenue in each period or the availability of government debt) can easily produce the empirical level of the nominal term premium. We extend their balanced budget fiscal model with wage rigidity and estimate the model with two versions of the output gap which is an exercise that is not conducted in Horvath et al. (forthcoming).

[11] This is the part of term premium which is a compensation for non-asset-specific risks.

3 The model

3.1 HOUSEHOLDS

Our model is based on the New Keynesian DSGE model of RS. The description of the households' and firms' optimization problem below closely follows RS.

The representative household maximizes the continuation value of its utility (V), which takes the Epstein-Zin form and follows the specification of RS:

$$V_t = \begin{cases} U(C_t, L_t) + \beta \left[E_t V_{t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) \geq 0 \\ U(C_t, L_t) - \beta \left[E_t (-V_{t+1})^{1-\alpha} \right]^{\frac{1}{1-\alpha}} & \text{if } U(C_t, L_t) < 0. \end{cases} \tag{1}$$

In equation 1, α is the Epstein-Zin curvature parameter. Epstein-Zin curvature captures the fact that highly risk-averse households demand early resolution of uncertainty. The representative household's utility maximization problem is subject to its flow budget constraint:

$$B_t + P_t C_t + T_t = W_t L_t + D_t + R_{t-1} B_{t-1}. \tag{2}$$

In equation (1), β is the discount factor. Utility (U) in period t is derived from consumption (C_t) and leisure ($1 - L_t$). E_t denotes expectations conditional on the information available at time t. L_t is hours worked. $W_t L_t$ is labor income, R_t is the gross interest rate on bonds, B_t, and D_t is dividend income. T_t represents taxes net of transfers.

To be consistent with balanced growth, RS impose the following functional form on U:

$$U(C_t, L_t) = \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-L_t)^{1-\chi}}{1-\chi}, \quad \varphi, \chi > 0. \tag{3}$$

where Z_t is an aggregate productivity trend and φ, χ, $\chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$, and the Frisch labor supply elasticity is given by $(1 - \bar{L})/\chi\bar{L}$, where $\bar{L}$ is the steady state of hours worked.

For the functional form in equation (3), RS derive the following relationship between the coefficient of relative risk-aversion ($CRRA$) and the curvature parameter α in the recursive utility function (1):

$$CRRA = \frac{\varphi}{1 + \frac{\varphi}{\chi}\frac{1-\bar{L}}{\bar{L}}} + \alpha \frac{1-\varphi}{1 + \frac{1-\varphi}{1-\chi}\frac{1-\bar{L}}{\bar{L}}}. \tag{4}$$

With preferences including habit formation in consumption[12], households fear uncertainty over consumption in the short run. EZ preferences allow for a separation between risk aversion and the IES. With EZ preferences, it is possible to increase the risk aversion of investors who are, therefore, more concerned about the volatility of their consumption path in the short, medium and long run.

3.2 FIRMS

There is a perfectly competitive sector that purchases the continuum of intermediate goods and turns them into a single final good using a CES aggregator of the following form:

$$Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{1}{1+\theta}} di \right]^{1+\theta}.$$

[12] See Jaccard (2014) for the merits of habit formation in both consumption and leisure.

The cost-minimisation problem of the representative perfectly competitive firm yields the demand schedule for intermediary firm *i*:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta}{\theta}} Y_t. \tag{5}$$

where θ is the net markup, $\frac{1+\theta}{\theta}$ is the elasticity of substitution among intermediary goods and the aggregate price index is defined as:

$$P_t \equiv \left[\int_0^1 P_t(i)^{-1/\theta} di\right]^{-\theta}.$$

Intermediary firms maximize their profits and face Calvo-style price-setting frictions. Accordingly, a $1-\xi$ fraction of firms can set their prices optimally in each period. Intermediate firm *i* produces output ($Y_t(i)$) using the technology

$$Y_t(i) = A_t[K_t(i)]^{1-\eta}[Z_t L_t(i)]^{\eta}. \tag{6}$$

3.3 AGGREGATION

Using the production function (equation 6) and the demand of firm *i* (equation 5) one can aggregate accross firms and derive the aggregate production function:

$$Y_t = \Delta_t^{-1} A_t [K_t]^{1-\eta}[Z_t L_t]^{\eta},\ 0 < \eta < 1, \tag{7}$$

where $K_t = Z_t \bar{K}$ is the aggregate capital stock ($\bar{K}$ is fixed), $L_t \equiv \int_0^1 L_t(i)di$ is aggregate labor, η is the share of labor in production, $\Delta_t \equiv \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\theta}{\theta\eta}} di\right]^{\eta}$ is price dispersion due to Calvo price-setting frictions. A_t is a stationary aggregate productivity shock,

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

where ε_t^A is an independently and identically distributed (iid) stochastic technology shock with mean zero and variance σ_A^2.

3.4 MONETARY AND FISCAL POLICY

The New Keynesian model is closed by a monetary policy rule (the so-called Taylor rule). We consider different Taylor rule specifications that are nested by the form in RS:

$$R_t = \rho_i R_{t-1} + (1-\rho)[R + \mathcal{I}\log\bar{\Pi}_t + g_\pi(\log\bar{\Pi}_t - \log\Pi_t^*) + g_y(Y_t - Y_t^*)/Y_t^*] + \varepsilon_t^i \tag{8}$$

where R_t is the policy rate, $\bar{\Pi}_t$ is a four-quarter moving average of inflation, and Y_t^* is the trend level of output yZ_t (where y denotes the steady-state level of Y_t/Z_t). Below, we also consider an alternative measure of the output gap whereby the trend level of output refers to the flexible-price version of output. Π_t^* represents the target rate of inflation, and ε_t^i is an iid shock with mean zero and variance σ_i^2. In the baseline version of the RS model without long-run inflation risks, the inflation target is constant ($\Pi_t^* = \Pi^*$ for all t). The choice of $\mathcal{I} = 1$ delivers the Taylor rule in RS. We also study interest rate rules with $\mathcal{I} = 0$ (in this case, the coefficient on the inflation gap is $1 + g_\pi$ to ensure that Taylor principle holds).

The four-quarter moving average of inflation ($\bar{\Pi}_t$) can be approximated by a geometric moving average of inflation:

$$\log\bar{\Pi}_t = \theta_\pi \log\bar{\Pi}_{t-1} + (1-\theta_\pi)\log\Pi_t, \tag{9}$$

where the choice of $\theta_\pi = 0.7$ ensures that the geometric average in equation (9) has an effective duration of approximately four quarters.

In one version of the RS model, the inflation target is time varying and has three properties: i) the inflation target is inertial ($\rho_{\pi^*}\Pi_{t-1}^*$), households adjust the inflation target upward (downward) when current inflation is higher (lower) than the inflation target ($\vartheta_{\pi^*}(\bar{\Pi}_t - \Pi_t^*)$) and is stochastic due to inflation target shocks ($\varepsilon_t^{\pi^*}$):

$$\Pi_t^* = \rho_{\pi^*}\Pi_{t-1}^* + \vartheta_{\pi^*}(\bar{\Pi}_t - \Pi_t^*) + \varepsilon_t^{\pi^*}, \quad \vartheta_{\pi^*} > 0, \tag{10}$$

where $\varepsilon_t^{\pi^*}$ is an iid inflation target shock with mean zero and variance $\sigma^2_{\pi^*}$. Note that equation (10) can also be described as an adaptive rule that captures a 'learning' of the target inflation.

Government spending follows the process

$$\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G, \;\; 0 < \rho_G < 1,$$

where $\bar{g}$ is the steady-state level of $g_t \equiv G_t/Z_t$, and ε_t^G is an iid shock with mean zero and variance σ_G^2. RS assume that government spending is financed through lump-sum taxes in each period, i.e., the government's budget is balanced. In section 8 of the paper we relax the lump-sum taxation assumption and introduce income taxes.

3.5 PRICING REAL AND NOMINAL ASSETS

This section is dedicated to provide formal definitions of the nominal, real and inflation risk premium, respectively. The price of a default-free, *n*-period zero-coupon bond that pays \$1 at maturity can be described with a recursive formula (see also RS (2012)):

$$p_t^{(n)} = E_t\{M_{t+1} p_{t+1}^{(n-1)}\}$$

where $M_{t+1} \equiv M_{t,t+1}$ is the stochastic discount factor; $p_t^{(n)}$ denotes the price of the bond at time *t* with maturity *n*; and $p_t^{(0)} \equiv 1$, i.e., the time-*t* price of \$1 delivered at time *t* is \$1. Given the functional forms in equation 1 and 3 the stochastic discount factor is given by:

$$M_{t,t+1} \equiv \left(\frac{C_{t+1}}{C_t}\right)^{-\varphi} \left[\frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}}\right]^{-\alpha}. \tag{11}$$

Equation 11 shows that the variation is the stochastic discount factor comes from two terms: the first is the standard ratio of marginal utilities (first fraction on the right-hand side), while the second fraction is due to Epstein-Zin curvature. When the model is approximated to the third-order, the Epstein-Zin term engineers conditional heteroskedasticity in model variables even if the driving shocks are homoskedastic.

To calculate the term premium, we need the bond price expected by the so-called risk-neutral investor who is rolling over a one-period investment for 10 years (in this case a bond with 10-year maturity). The risk-neutral bond price can be expressed through the expectations hypothesis of the term structure:

$$\widehat{p_t}^{(n)} = e^{-i_t} E_t \widehat{p}_{t+1}^{(n-1)} \tag{12}$$

where, again, $\widehat{p_t}^{(0)} \equiv 1$. Equation (12) is another recursion that states that the current-period price of the bond is the present discounted value of the next-period bond price and the discount factor is the risk-free rate rather than the stochastic discount factor.

The continuously compounded yield to maturity of the *n*-period, zero-coupon nominal bond is defined as

$$i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)}.$$

The implied nominal term premium is defined as the difference between the yield expected by the risk-averse investor ($i_t^{(n)}$) minus the yield expected by the risk-neutral investor ($\widehat{\iota}_t^{(n)}$):

$$NTP_t^{(n)} = i_t^{(n)} - \widehat{\iota}_t^{(n)}$$

In Section 8 we also consider two alternative measures of the NTP. The first is the slope of the nominal term structure which we define as the difference between the ten-year bond yield and the three-month bond yield. The second is the excess holding period return which is the difference between the yield on a fourty-quarter bond which is bought in quarter 39 and sold in quarter 40 minus the yield on a three-month bond.

Similarly, we can use the *real* stochastic discount factor to price real bonds held by the risk-averse and risk-neutral investors. Again, the difference between the risk-averse and the risk-neutral yields in the case of real bonds delivers the real term premia

($RTP_t^{(n)}$). Intuitively, the real term premia is a compensation for real risks (due to negative realisations of the technology process). Below we discuss that the real risks are almost completely eliminated by a high-coefficient on the output gap in the Taylor rule.

Andreasen (2012) shows that the inflation risk premium ($IRP_t^{(n)}$) can be approximated to the third-order as the difference between the nominal ($NTP_t^{(n)}$) and real term premia ($RTP_t^{(n)}$):

$$IRP_t^{(n)} = NTP_t^{(n)} - RTP_t^{(n)}.$$

4 Calibration and solution method

Calibration is available in Table 2 and follows the baseline parameter values of RS. Our models are solved by a third-order Taylor series approximation using Dynare (Adjemian et al. 2011). To study the standard deviation of the NTP, third-order approximation is needed. The means, however, are quite similar in magnitude when approximation is either second- or third-order. It is important to note that this literature uses a high risk-aversion coefficient to generate a reasonable level of the nominal term premia (see Andreasen (2012), RS (2008, 2012), Li and Palomino (2014), Hsu et al. (2020), and Kliem and Meyer-Gohde (2017)). Due to the Epstein-Zin preferences, there is separation between risk aversion and the IES, meaning that high risk aversion is not coupled with a counterfactually low IES, which in our calibration is 0.5 and well in line with the empirical evidence reported by Havranek et al. (2015).

Table 2
Calibration

γ	1.0025	φ	2	ρ_i	0.73	ρ_A	0.95
$\widetilde{\beta}$	0.99	χ	3	g_π	0.53	ρ_G	0.95
δ	0.02	CRRA	75	g_y	0.93	σ_A	0.005
$\bar{L}$	1/3	η	2/3	Π^*	1	σ_G	0.004
K/Y	10	θ	0.2	ρ_{π^*}	0.99	σ_i	0.003
G/Y	0.17	ξ	0.75	σ_{π^*}	0.0005		
ε	6			ϑ_{π^*}	0.01		

Notes: G/Y is the government spending-to-GDP ratio, K/Y is the share of fixed capital in GDP, and δ is the depreciation rate of fixed capital. ε is the elasticity of substitution among intermediary goods, implying a net markup of twenty percent ($\theta = 0.2$). The rest of the parameters are explained above.

5 Results

In this section we explore the effect of the specification of the Taylor rule on nominal, real and inflation risk premia. Below, we devote a separate section to the welfare evaluation of interest rate rules. Table 3-5 contain the mean of NTP, RTP, IRP, and the conditional welfare (W) for six different versions of the interest rate rules that we consider. We also explore the robustness of our results to two different versions of the output gap. In particular, columns 1-4 contain the simple version of the output gap defined as the deviation of the sticky-price output from its deterministic steady state, while in columns 5-8, it is used relative to the flexible-price output.[13] In this section we focus on the mean of NTP as well as its components: RTP and IRP. In section 8, however, we estimate an extended version of the models on US data and match not only the mean but also the standard deviation of the nominal term premium.

Table 3
Taylor rule specifications when the coefficient on the output gap is closer to zero: Low coefficient on the output gap (no long-run inflation target risks)

Case	Meaning	simple output gap				flexible output gap			
		NTP	RTP	IRP	W	NTP	RTP	IRP	W
1	Π_t	0.59	0.15	0.44	0.82	0.49	0.18	0.31	0.73
2a	Π_{t+1}	0.64	0.15	0.49	0.87	0.52	0.17	0.35	0.75
2b	$\bar{\Pi}_t$	0.42	0.06	0.36	0.77	0.37	0.11	0.25	0.71
3a	1 w/ smoothing	0.45	0.10	0.35	0.80	0.38	0.14	0.24	0.74
3b	2b w/ smoothing	0.35	0.05	0.30	0.78	0.31	0.10	0.21	0.72
4	RS rule	0.29	0.04	0.24	0.71	0.28	0.06	0.22	0.70

Notes: TR means Taylor rule. NTP, RTP and IRP denote the nominal, real and inflation risk premium, respectively. IRP is calculated as NTP-RTP. W stands for conditional welfare expressed in consumption equivalents. In all cases, $\mathcal{I}$ =0 and g_y = 0.07 unless indicated otherwise. In this setup, there are no long-run inflation target risks ($\vartheta_{\pi^*} = \rho_{\pi^*} = \varepsilon_t^{\pi^*} = 0$). Case 4 corresponds to the RS model with $g_y = 0.07$. In line with RS, we express inflation and interested rates in annual terms in case 4.

5.1 INTEREST RATE RULE WITHOUT A TIME-VARYING INFLATION TARGET AND LOW COEFFICIENT ON THE OUTPUT GAP

Table 3 shows results for the case of low coefficient on the output gap: 0.07 (see the estimate of Clarida et al. (1998) in Table 1). In Tables 3–4, there is no time-varying inflation target (captured by setting $\vartheta_{\pi^*} = \rho_{\pi^*} = \varepsilon_t^{\pi^*} = 0$ in equation (8)). To make our points clear we first discuss the cases when there is no inertia in inflation and in the policy rate (captured by setting $\theta_\pi = \rho_i = 0$).

5.1.1 NO INERTIA IN INFLATION AND IN THE POLICY RATE

First, we consider the simplest Taylor rule without interest rate smoothing and in which monetary policy reacts to current-period inflation (case 1). When we use a simple definition of the output gap, the IRP is higher (53 basis points) than with the

[13] Note that we do not report macro moments (and additional financial moments) for each case due to space constraint (this would be a twelve-page long table for the six Taylor rule types and the two output gap definitions) but the results are available upon request. However, the full picture is provided for our estimated extended model in the last section of the paper.

flexible-price version of the output gap (26 basis points). With the simple output gap, nominal risks are higher but real risks are lower compared to the case of a flexible-price output gap (the same pattern is true for the remaining interest rate rule specifications). To shed light on the implications of the two different conceptualizations of the output gap, we consider the reaction of the output gaps to positive technology shocks.

The simple output gap reports overheating of the economy (positive gap) in response to a positive technology shock (sticky-price output rises relative to the steady state). Whereas the flexible price version arrives at exactly the opposite conclusion because flexible-price output rises more than sticky-price output in the event of a technology shock, leading to a negative output gap. In the latter case, therefore, both inflation and output are below their reference levels, and thus, inflation risks are low even if the Taylor rule prescribes a decline in real rates to support recovery. Moreover, the flexible-price version is directly influenced by the technology shocks, making the output gap more volatile and increasing real risks (all RTP values are higher in the case of the flexible-price output gap).

In the inflation-targeting system some central banks, however, react not to current inflation but to expected future inflation. Hence, we replace current inflation with expected inflation, π_{t+1} (case 2a) in the Taylor rule. In the case 2a, inflation risks are somewhat higher than in case 1 because there is no reaction to inflation at time *t*, thus automatically attributing higher weight to output gap stabilization at time *t* and an increase in inflation risks. Benhabib et al. (2003) assert that an interest rate rule that contains future expected inflation can lead to self-fulfilling equilibria in which arbitrary changes in households' expectations about the future evolution of the economy can affect the real variables in the model.

5.1.2 INTRODUCING INERTIA IN INFLATION

In US data inflation is highly persistent which we mimick by introducing inertia in inflation. Inflation inertia is captured through an average of current and past inflation (denoted by $\bar{\pi}_t$). Time-*t* shocks affect current inflation more than the average of current and past inflation and, thus, have reduced real and nominal effects. As a result, inflation and output-gap stabilization are more successful, thereby resulting in lower inflation and real risks (see case 2b, where NTP, RTP and IRP are much lower than in cases 1 and 2a.). Indeed, previous papers such as Carlstrom and Fuerst (2000) report that an interest rate rule that reacts aggressively to past inflation (or an average of current and past inflation) helps to anchor inflationary expectations and to avoid real indeterminacy. Further, they argue that making the interest rate rule sufficiently backward looking can help avoid self-fulfilling prophecies.

5.1.3 INTRODUCING INERTIA IN THE NOMINAL INTEREST RATE

To capture the gradualism in monetary policy we introduce interest rate smoothing. In cases 3a and 3b, we extend cases 2a and 2b, respectively, with interest rate smoothing, which helps to mitigate nominal and real risks through the delayed monetary policy response. To illustrate this, consider a negative technology shock that leads to a rise in the real interest rate and makes the households consume less according to the logic of the Taylor rule. The rise in the real interest rate is gradual under interest rate smoothing, and thus, the household can adjust its labor supply more in a given period to insure itself against the negative effects of the shock (less consumption).

The last row of Table 3 displays the results when the RS formulation of the interest rate rule is adopted (see case 4). In particular, we employ the Taylor rule specification of RS, who define the inflation rate in annual terms such that the coefficient, g_π on the inflation gap $\log(\bar{\Pi}_t/\Pi_t^*)$ is smaller and would not satisfy the Taylor principle. To avoid this problem, RS introduce another inflation term in the Taylor rule (in our notation, this is invoked by setting $\mathcal{I}$ =1). The separate responses to inflation (see the term with $\mathcal{I}$ =1) and the inflation gap further reduce nominal risks, suggesting that the standard deviation of inflation is more penalized in this setup.

5.2 INTEREST RATE RULE WITHOUT A TIME-VARYING INFLATION TARGET AND HIGH COEFFICIENT ON THE OUTPUT GAP

More recent estimates of the output gap for the United States are closer to one than to zero (see, e.g., Coibion and Gorodnichenko, 2011). In Table 4, we recalculate the above models with a high coefficient on the output gap. The previous patterns are confirmed but we observe one striking difference between the results in Table (3) and (4).

The higher output gap coefficient systematically generates higher inflation risks in case of the simple definition of the output gap. This is because there is no divine coincidence in the model i.e. the standard deviations of inflation and the output gap cannot be simultaneously reduced. When monetary policy assigns a higher priority to stabilizing fluctuations in the output gap, inflation risks receive relatively less attention and automatically rise.

In the case of flexible output gap the picture is different. With higher coefficient on the output gap the real risks (measured by the real term premium) in the economy are slightly higher due to the direct effect of the technology shock on the flexible price output as argued above whereas the nominal term premium is lower. Because the nominal term premium is the sum of the real term premium and the inflation risk premium then it must be the case that the inflation risk premium component has shrinked. Note that the gap in absolute terms is much smaller in case of flexible price version because the reference level of the output (the flexible price benchmark) is closer to the sticky price level and, thus, the increased weight attached to output gap stabilisation (in the form of a higher output gap coefficient) does not imply a large reduction in the significance of the inflation stabilisation objective. Hence, the flexible price output gap concept produces an interesting and unexpected pattern: a larger weight attributed to output-gap stabilisation leads to somewhat more real risks but a decrease in inflation risks.

Table 4
Taylor rule specifications when the coefficient on the output gap is close to one: High output-gap coefficient (no long-run inflation target risks)

Case	Meaning	simple output gap				flexible output gap			
		NTP	RTP	IRP	W	NTP	RTP	IRP	W
1	Π_t	1.30	-0.01	1.31	2.52	0.41	0.19	0.22	0.69
2a	Π_{t+1}	1.37	-0.02	1.39	2.81	0.43	0.19	0.23	0.69
2b	$\bar{\Pi}_t$	0.93	-0.16	1.09	2.13	0.36	0.16	0.19	0.68
3a	1 w/ smoothing	1.00	-0.13	1.13	2.20	0.34	0.16	0.17	0.69
3b	2b w/ smoothing	0.76	-0.22	0.98	1.95	0.30	0.15	0.16	0.68
4	RS rule	0.43	-0.08	0.51	0.95	0.29	0.08	0.21	0.69

Notes: NTP, RTP and IRP denote the nominal, real and inflation risk premium, respectively. W measures the conditional welfare cost. IRP is calculated as NTP-RTP. In all cases, $\mathfrak{I}$ =0 unless indicated otherwise. In this setup, there are no long-run inflation target risks ($\vartheta_{\pi^*} = \rho_{\pi^*} = \varepsilon_t^{\pi^*} = 0$). Case 4 corresponds to the RS model, which expresses inflation and interested rates in annual terms.

5.3 DOES A TIME-VARYING INFLATION TARGET WITH 'LEARNING' GENERATE INFLATION RISKS?

In this section, we revisit the question of whether the time-varying inflation target with 'learning' can be a source of inflation risks. The time-varying inflation target ($\rho_{\pi^*} > 0$) with 'learning'[14] is captured by equation (10), which is similar to that in Gürkaynak et al. (2005) and is a well-known feature used to induce uncertainty over the current and future inflation target.[15] In this simple framework, there is learning of the inflation target whenever the average of current and past inflation differs from current-period inflation target, i.e., $(\bar{\Pi}_t - \Pi_t^*) \gtrless 0$ and $\vartheta_{\pi^*} > 0$ governs the degree to which the difference $(\bar{\Pi}_t - \Pi_t^*)$ feeds back into the current-period inflation target.

We explore the relevance of time-varying inflation target in the case of a high output gap coefficient, which is our baseline calibration. When comparing the model without a time-varying inflation target with learning and that including these features (see Tables 4 and 5, respectively), it is apparent that the time-varying inflation target with learning has marginal effects on inflation risk premia for both versions of the output gap and for each specification of the Taylor rule (consistent with the findings

[14] Note that our adaptive rule serves as a short-cut to a full-fledged model with learning.

[15] In the United States, the Federal Reserve did not make any explicit statements about future inflation targets until at least 2012.

of Hördahl et al. (2008)). However, it is an effective tool to increase the standard deviations of nominal variables (inflation and most of the financial variables—not reported in the tables) and the standard deviation of the nominal term premium (in line with the findings of RS).

Table 5
Taylor rule specifications when the coefficient on the output gap is close to one: High output gap coefficient and long-run inflation target risks with 'learning'

Case	Meaning	simple output gap				flexible output gap			
		NTP	RTP	IRP	W	NTP	RTP	IRP	W
1	Π_t	1.53	-0.00	1.53	8.66	0.39	0.20	0.19	6.08
2a	Π_{t+1}	1.61	-0.02	1.63	9.05	0.40	0.20	0.21	6.11
2b	$\bar{\Pi}_t$	1.05	-0.16	1.21	8.18	0.27	0.18	0.10	6.11
3a	1 w/ smoothing	1.14	-0.13	1.27	8.25	0.26	0.17	0.09	6.08
3b	2b w/ smoothing	0.82	-0.21	1.03	7.96	0.18	0.17	0.00	6.13
4	RS rule	0.46	-0.09	0.55	1.02	0.30	0.08	0.23	0.73

Notes: NTP, RTP and IRP denote the nominal, real and inflation risk premium, respectively. W measures conditional welfare cost. IRP is calculated as NTP-RTP. In all cases, $\mathfrak{I}$ =0 unless indicated otherwise. In this setup, long-run inflation target risks are included ($\vartheta_{\pi^*} > 0$, $\rho_{\pi^*} > 0$, $\varepsilon_t^{\pi^*} > 0$). Case 4 corresponds to the RS model with a time-varying inflation target that is subject to a 'learning' process. In case 4, inflation and interest rates are in annual terms, in line with RS.

6 Welfare analysis

We report the welfare cost (W) in terms of consumption equivalents relative to deterministic steady state in the tables above. More specifically, we follow Schmitt-Grohe and Uribe (2007) and define consumption equivalents as the percentage compensation in the consumption process associated with the policy rule necessary to make the level of welfare under the steady state identical to that under the considered policy. Thus, a positive figure indicates that welfare is higher under the deterministic steady state than under the considered policy.

Schmitt-Grohe and Uribe (2007) report both conditional and unconditional welfare. As the sign of the two welfare measures is the same (these results are reported in appendix A) we report only one of them (the conditional welfare measure) to keep the clarity of the exposition. In the following, we provide an example for the interpretation of welfare measures: for instance, in the model with RS rule (see the last row of Table 3) households are missing 0.72% of their life time consumption to reach the welfare they would enjoyed in deterministic steady state and in the model from the first row of Table 3 the welfare is lower by 0.91% of households life time consumption.

We make the following observations regarding the welfare measures across the various interest rate rules. First, the comparison of Tables 3 and 4 reveals that welfare costs are higher in the simple gap model than in the flexible price gap model. This can be explained as follows: first, the simple gap model measures the output gap as the difference between the sticky price output and steady-state output. The latter difference can be large and definitely higher than the difference between sticky and flexible price outputs in the flexible gap version. The higher gap translates into the higher associated welfare losses.

Second, the comparison of the simple gap and flexible gap parts in Table 3 indicates that a higher output gap coefficient in the simple gap model implies higher welfare costs whereas the flexible price gap model is not very sensitive to the size of the output gap coefficient (see Tables 3 and 4).

Third, Table 5 shows that the simple output gap model with high output gap coefficient and long-run inflation target risks imply enormous welfare losses. Still, the model with long-run inflation target risks are not affected by the high output gap coefficient when output gap is defined as the flexible price measure (see, again, Table 5).

Finally, we note that our results point to the high unconditional correlation of nominal term premia with all measures of welfare (e.g. one can compare Tables 3 and 4 to see that a higher mean of the nominal term premium is associated with higher welfare cost and a higher minimized value of the loss function).

7 Real or nominal rigidities and the term-premia

In this section we argue that the introduction of real rigidities or price rigidity help explain bond risk-premia with Epstein-Zin preferences. Previous studies using production economies with sticky prices and habits in the felicity function claim that the introduction of price rigidity does not help to explain asset prices. This is because it generates lower equity and bond premia (see de Paoli et al. (2010), Hördahl et al. (2008) and Rudebusch and Swanson (2008)) in a world driven by either temporary productivity or monetary policy shocks. Previous papers with recursive preferences feature price rigidity (see, e.g., RS (2012)) but do not analyze the effect of introducing price rigidity (or higher price rigidity) on finance moments such as the slope of the term structure or nominal/real term premia. First, we argue that real rigidities such as firm-specific capital is observationally equivalent to higher price stickiness in the model and help explain bond premia.

Table 6
Comparison of some model moments with and without firm-specific capital

	No firm-specific capital (less rigid prices)	Firm-specific capital (more rigid prices)
NTP	0.3137	0.4008
corr(C, π)	-0.8209	-0.8703
std(L)	0.9338	1.4238
std(Δ)	0.0650	0.2566

Notes: In the table NTP, corr(C,π), std(L), and std(Δ) denote the nominal term premium, the correlation between consumption and inflation, and the standard deviation of hours worked and price-dispersion, respectively. NTP, std(L), and std(Δ) are all measured in percentages. In this table, we use the baseline calibration. We abstract from reporting other moments because they are quite similar across the two versions of the model. It is easy to see by observing equation (7) that firm-specific fixed capital can be eliminated by the choice of $\eta = 1$ (then, the result is a constant-returns-to-scale production function with labor).

7.1 THE EFFECTS OF FIRM-SPECIFIC CAPITAL

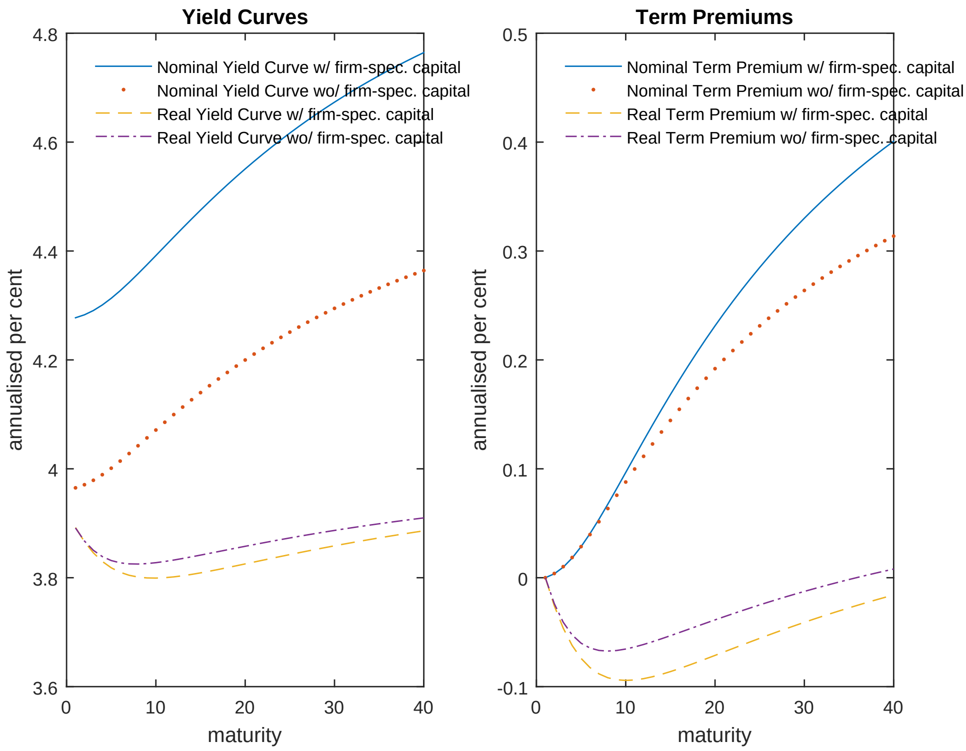
Our model with recursive utility better describes bond market statistics (e.g., the nominal term premium) when there is price rigidity or another real friction is added such as firm-specific inputs, which work similarly to increased price stickiness. Such real frictions can be firm-specific labor (capital) markets or Kimball demand. In this paper, similar to RS, the assumption of firm-specific fixed capital (or simply firm-specific capital for short) is proposed as a real friction and can be interpreted as a model that includes capital with an infinite adjustment cost.

Technically, the inclusion of firm-specific capital leads to lower slope of the New Keynesian Phillips curve.[16] A lower slope of the New Keynesian Phillips curve can be also achieved by the higher degree of price rigidity (or higher price-adjustment costs in a Rotemberg setting). Therefore, the effects of higher price rigidity and the inclusion of firm-specific capital are observationally equivalent.

In Figure 1, we gauge the effect of firm-specific capital on the nominal and real yield curves (left panel) and nominal term premia (right panel). We find that firm-specific capital increases inflation risks, especially at longer maturities.[17] Firm-specific capital

[16] To the first-order Taylor series approximation, it can be shown that the assumption of firm-specific capital or labor injects another term into the denominator of the slope of the Phillips curve. In the technical appendix of Horvath et al. (2020), we show how the various assumptions about the factor market affect the slope of the Phillips curve.

Figure 1
The effect of firm-specific capital on bond yields and term premia



serves as a vehicle for households to insure against real risks but operates as leverage in the case of inflation risks.

We begin with the implications of firm-specific fixed capital for real risks (measured by the real term premium). Firm-specific capital operates as a precautionary savings effect, driving down real yields as a result of higher savings (see the left panel of the figure, where real yields are lower given firm-specific capital at all maturities). The higher precautionary savings effect is mirrored by a lower real term premium in the case of firm-specific capital relative to the case of no firm-specific capital for all maturities except those shorter than a year.

Next, we investigate why the introduction of price stickiness (a higher degree of price rigidity due to the introduction of firm-specific fixed capital) can contribute to an increase in inflation risks (measured as the difference between the nominal and real term premia).

7.2 PRICE STICKINESS WITH EPSTEIN-ZIN PREFERENCES

The earlier literature (see, e.g., Hördahl et al. (2008)) featuring preferences with habits concludes that higher price rigidity causes lower premium on risky assets such as government bonds and equities. Below we argue that this conclusion is reversed with Epstein-Zin preferences. In the case of preferences with habits, the representative consumer associates the highest price of risk with shocks that generate business cycle fluctuations. With Epstein-Zin recursive preferences, however, the prices of fluctuations at the lowest frequencies are orders of magnitude higher so agents fear shocks which induce permanent or highly persistent fluctuations in consumption growth (see Dew-Becker and Giglio (2016)). However, technology shocks are not the only possible source of persistence in the economy.

To understand why price rigidity contributes to the price of risk as well as term premia we compare the economy under flexible and rigid prices with Epstein-Zin preferences. Under flexible prices, the adjustment in real variables is larger, less persistent and

[17] See the figure, where nominal yields are higher given firm-specific capital for all maturities (left panel), and the nominal term premium for maturities of at least ten quarters is also higher with firm-specific capital (right panel)

price adjustment is immediate in response to changes in technology. On the other hand, rigid prices imply larger persistence in real variables because many of the firms are not able to reset their price for extended period of time (see, e.g., Ireland (2004)). With Epstein-Zin preferences long-term persistent fluctuations carry higher price of risks and therefore mechanisms that induce more protracted variations in economic indicators generate higher risk-premia in the yields of long-term bonds.

In Table 6, we collect those variables with moments that are remarkably different between the model with more (firm-specific capital) and less (the absence of firm-specific capital) price rigidity. In the Calvo model of price rigidity, there is price-dispersion, meaning that one unit of labor produces less than one unit of output.[18] When price dispersion is higher due to a longer duration of price stickiness, the correspondence between input and output is even less effective, and thus, labor and consumption are more volatile. The standard deviations of labor and price dispersion are higher for the model with firm-specific fixed capital (or, equivalently, higher in the model with a higher degree of price stickiness); see the third and fourth rows of Table 6.

Due to the higher price dispersion, a higher average duration of price stickiness reduces the effectiveness of labor supply at insuring against negative shocks, and the household requires a higher premium to hold a risky asset, such as the nominal bond in our example. Nominal bonds are risky due to the negative comovement between consumption and inflation (in the event of a negative realization of technology, consumption is low but inflation is high, thus reducing the real return on the bond). With firm-specific capital (the case of higher price stickiness), this negative comovement is stronger and the NTP is higher (0.40) relative to a model without it (0.32) (see the second row of Table 6).

[18] This is only for the purpose of illustrating the effect of price dispersion. Here, we assume only labor in production and that technology is fixed at one.

8 Model estimation

In the previous sections we kept the model simple intentionally so that we can track how the different versions of the Taylor rule affect the mean of the nominal term premium. However, our previous research (see Horvath et al. (forthcoming)) finds that a richer fiscal setup where government spending is financed by labor income taxes[19] and the assumption of real wage rigidity helps the model matching macro and finance data jointly. Hence, we extend the model of RS with labor income taxation and wage rigidity (the additional equations can be found in Appendix B) and estimate this richer model on US data over 1961-2007[20] by GMM and SMM.[21]

8.1 THE GMM AND SMM ESTIMATORS

The GMM and SMM estimators are shorty summarized as follows. Letting θ denote the structural parameters, the GMM estimator is given by:

$$\arg\min_{\theta\in\Theta}\left(\frac{1}{T}\sum_{t=1}^{T}q_t - E(q_t(\theta))\right)' W\left(\frac{1}{T}\sum_{t=1}^{T}q_t - E(q_t(\theta))\right). \tag{13}$$

In equation (13), W is a positive definite weighting matrix, $\frac{1}{T}\sum_{t=1}^{T}q_t$ are data moments and $E(q_t(\theta))$ are moments computed from the model. We follow a conventional two-step procedure to implement GMM. In the first step, we set $W_T = diag(\widehat{S}^{-1})$ to obtain $\widehat{\theta}^{(1)}$, where $\hat{S}$ denotes the long-run variance-covariance matrix of $\frac{1}{T}\sum_{t=1}^{T}q_t$ when centered around its sample mean. In the second (final) step, we obtain $\widehat{\theta}^{(2)}$ using the optimal weighting matrix $W_T = diag(\widehat{S}^{-1}_{\widehat{\theta}^{(1)}})$, where $\widehat{S}^{-1}_{\widehat{\theta}^{(1)}}$ denotes the long-run variance of our moments re-centered around $E\left(q_t(\widehat{\theta}^{(1)})\right)$. The long-run variances in both steps are produced with the Newey-West estimator using five lags, and our results are robust to the inclusion of, e.g., ten lags. In case of the GMM estimator it is possible to compute analytical expressions for $E(q_t(\theta)$ but for the SMM estimation it is replaced by a simulation-based estimate. Hence, in summary, the GMM and SMM estimators update the parameters estimates such that the difference between data- and model-based moments reduces to a minimum.

8.2 THE INPUTS FOR THE GMM AND SMM ESTIMATION

We use the following seven times series to estimate the model: real consumption growth, hours growth, real wage growth, inflation, the growth rate of labor tax revenue divided by GDP, the slope of the term structure (the difference between the forty- and one-quarter Treasury yield) and the nominal term premium. We describe the data in more detail in the Appendix C. In line with Andreasen et al. (2020) and Bretscher et al. (2020), we consider three types of unconditional moments for the GMM and SMM estimation: i) sample means $m_1(y_t) = y_t$, contemporaneous covariances $m_2(y_t) = vech(y_t y_t')$, and own autocovariances $m_3(y_t) = \{y_{i,t}y_{i,t-k}\}_{i=1}^{n_y}$ for $k = 1$ and $k = 5$. The total set of moments used in the estimation are, therefore, given by $m(y_t) = [m_1(y_t)\ m_2(y_t)\ m_3(y_t)]'$. Hence, we use seven means and standard deviations, seven first- and fifth-order autocorrelations and twenty-one covariances (based on the symmetric variance-covariance matrix) to estimate the model in total.

Note that some of the parameters and steady-state quantities are not estimated but calibrated. The steady-state inflation is zero ($\Pi^* = 1$). The government spending-to-GDP ratio is twenty percent that is roughly in line with post-war US data. The

[19] In this paper, we assume that government purchases are financed by labor income taxes where the government spending is balanced in each period. In Horvath et al. (forthcoming) we also discuss the empirically more realistic case of time-varying government debt.

[20] We focus on data before the great recession to avoid complications posed by the fact that the US policy rate reached its lower bound at the end of 2008.

[21] To conduct GMM and SMM estimations, we use the toolboxes of Andreasen et al. (2018), which allows us to use a third-order approximation of the model.

yearly government debt-to-GDP ratio (γ_b) is calibrated to be sixty percent. The steady-state capital-to-GDP ratio is chosen to be ten as in RS and the depreciation rate is ten percent per annum. The trend growth rate of the economy (γ) is fixed at the rate of one percent per annum.

8.3 RESULTS FROM THE GMM AND SMM ESTIMATION

We estimate two versions of our extended model. The first version employs the simple output gap which is defined as the deviation of the sticky-price output from its steady-state level. The second version of the output gap, which is used in the estimation, replaces the steady-state output with the flexible-price version of output. We present the parameter estimates in Table 7. The first two columns show GMM-based estimates while the last two columns are based on SMM. Below each estimate we report the standard error of the estimate in percent.

We find that the estimated parameters are roughly similar across the two model variants (simple and flexible price output gap) and across the estimation methods (GMM and SMM) in line with Andreasen et al. (2018).[22] The curvature parameter of the recursive utility, α, is estimated with a high standard error as in Andreasen et al. (2018) and Bretscher et al. (2020). The elasticity of substitution among goods is estimated to be four ($\varepsilon = \frac{1+\theta}{\theta}$) which is in the lower end of the empirical estimates (see Bernard et al. (2010)). Perhaps surprisingly, the interest rate smoothing in the Taylor rule and inflation smoothing is estimated to be lower than in RS. This can be explained through our fiscal extension which is more powerful (i.e., it produces higher inflation risk-premium) when inflation responds to a higher extent in each period such that inflation and the short-term nominal interest rate are less inertial. The rest of the parameter estimates are more in line with the ones in Andreasen et al. (2018).

8.4 MACRO AND FINANCE MOMENTS FROM THE ESTIMATED MODELS

Macro and finance moments (means and standard deviations) are calculated using the estimated parameters in the previous table and are available in Table 8. The macro moments reported include the standard deviation of consumption growth, real wage, hours worked, one-quarter and 10-year nominal interest rate, one-quarter real interest rate and inflation. The finance moments consist of the mean and standard deviation of the nominal term premium, the slope of the term structure ($R^{(40)} - R$), and the excess holding period return ($X^{(40)}$). The first column contains unconditional first and second moments based on US data 1961-2007. The second and third columns contain our two model versions using parameters from the GMM estimates. The last two columns contains the same models which are calibrated with the SMM estimates.

We make the following observations. The standard deviations of several real and nominal variables such as labor, real wage, short- and long-term interest rates and inflation are counterfactually low in case of the flexible price version of the output gap. This is well in line with our argument before (see section 4.1): inflation risks are smaller in the case of flexible price output gap because in that case the output gap is smaller or even negative[23] so that the relative weight attributed to stabilising the output gap is smaller and there is more space for inflation stabilisation. Hence, the variability of inflation (and the rest of the real and nominal variables) as well as the variability of the nominal term premium will be lower when the interest rate rule contains the flexible price version of the gap. The alternative measures of the NTP (the slope and the excess holding period return) have the same features as the NTP: their standard deviations are somewhat smaller in the flexible price gap case. The consumption growth is, however, more volatile in the flexible output gap model due to the appearance of technology shocks through the flexible price output in the Taylor rule. As a result the flexible gap model produces more real risks than the simple gap model (the RTP [not reported in the current table] which is given by difference between the mean of NTP and the IRP is also higher in the flexible gap model). Overall, we can say that the simple output gap version of the model generates better fit of the model in term macroeconomic and financial moments relative to the flexible gap model.

[22] Note that we estimated our models with three shocks: technology, monetary and government spending shocks. The results differ to small extent after adding inflation-target shock as well (these results are not reported in the paper but available upon request).

[23] In the case of a positive technology shock, for instance, the flexible price output might increase more than the sticky price output so the output gap may turn to negative even when the economy is experiencing a supply-side improvement.

Table 7
GMM and SMM estimates of the models

Parameters and steady-states	GMM Simple output gap	GMM Flexible price output gap	SMM Simple output gap	SMM Flexible price output gap
Household				
$\tilde{\beta}$	0.9982 0.0024	0.9982 0.0015	0.9981 0.0171	0.9981 0.0112
φ	2.0049 0.1	1.9997 0.34	2.0165 0.28	1.9812 0.42
χ	2.7125 0.13	2.7634 0.24	2.7142 0.32	2.7734 0.41
α	−118.1256 35.62	−115.4638 46.27	−118.7602 37.71	−116.6787 44.63
$\bar{L}$	0.3882 0.00062	0.3666 0.00042	0.3323 0.0017	0.3315 0.0038
CRRA (implied)	75.73	74.87	75.82	74.99
Firm				
η	0.5996 0.0039	0.5968 0.0062	0.5981 0.0052	0.5992 0.0083
ξ	0.8003 0.00024	0.8007 0.00044	0.8011 0.0031	0.8006 0.0025
μ	0.9611 0.17	0.9262 0.52	0.9513 0.58	0.9301 0.36
ε	4.0211 0.031	3.9502 0.022	4.0140 0.039	3.9101 0.062
Monetary Policy				
ρ_i	0.5334 0.0001	0.5318 0.0021	0.5261 0.0049	0.5712 0.0073
g_π	0.5197 0.0005	0.5257 0.0028	0.5236 0.0139	0.5361 0.0163
g_y	0.8427 0.0009	0.8254 0.0029	0.8224 0.0068	0.8327 0.0073
θ_π	0.3541 0.0009	0.3592 0.0029	0.3551 0.0037	0.3554 0.0048
Shock processes				
ρ_a	0.9719 0.0016	0.9366 0.0097	0.9513 0.0068	0.9536 0.0047
ρ_g	0.9801 1.51	0.9829 1.32	0.9826 1.48	0.9872 1.93
σ_a	0.0055 0.0038	0.0054 0.0089	0.0053 0.0079	0.0054 0.0061
σ_g	0.0142 0.0031	0.0131 0.0029	0.0125 0.0038	0.0136 0.0079
σ_i	0.0035 0.34	0.0034 0.21	0.0031 0.46	0.0042 0.27

Notes: The numbers below the parameter estimates denote the standard error of the estimate as a percent. The implied CRRA parameter is given by equation (4).

Table 8
Moments from the models

Unconditional Moment	US data 1961-2007	GMM Simple gap	GMM Flex. price gap	SMM Simple gap	SMM Flex. price gap
SD(dC)	2.78	2.83	4.36	2.52	4.50
SD(L)	0.80	0.64	0.14	0.60	0.18
SD($d(W/P)$)	0.97	0.79	1.33	0.80	1.25
SD(π)	2.52	2.29	0.62	2.08	0.57
SD(R)	2.71	2.62	1.68	2.38	1.67
SD(R^{real})	2.30	0.77	1.12	0.70	1.16
SD($R^{(40)}$)	2.41	1.83	0.87	1.63	0.80
MEAN($NTP^{(40)}$)	1.72	1.30	1.26	1.13	1.26
SD($NTP^{(40)}$)	1.21	0.27	0.13	0.22	0.17
MEAN($R^{(40)} - R$)	1.43	1.14	1.17	0.99	1.17
SD($R^{(40)} - R$)	1.33	1.07	0.90	0.99	0.95
MEAN($X^{(40)}$)	1.76	1.16	1.16	1.01	1.16
SD($X^{(40)}$)	23.43	8.15	5.57	7.35	5.57
MEAN($IRP^{(40)}$)	0.83	1.16	0.54	1.02	0.54
CORR(dC, π)	-0.34	-0.13	-0.18	-0.13	-0.19

Notes: MEAN, SD and CORR denote the unconditional mean, standard deviation and correlation, respectively. $NTP^{(40)}$=nominal term premium on a 40-quarter bond, $R^{(40)} - R$ is the slope, and $X^{(40)}$ is the excess holding period return for a 10-year bond. Moments are calculated using parameters estimated with GMM and SMM on US data for 1961-2007.

9 Concluding remarks

We use a macro-finance New Keynesian model and show that systematic monetary policy i.e. the specification as well as the size of the inflation and output gap coefficients in the Taylor rule have substantial effects on the amount of inflation risks. When monetary policy reacts to future inflation rather than current inflation, nominal risks are higher as current period inflation is not mitigated by the Taylor rule. In contrast, when current inflation is replaced with an average of past and current inflation, nominal risks are lower. The latter is an example of inflation smoothing which reduces the changes in the policy rate and the overall volatility of the economy in response to shocks.

We find that interest rate smoothing reduces the reaction of monetary policy in a given period such that households can adjust more to negative shocks by increasing their labor supply before monetary policy takes full effect (implemented over a longer period). When the output gap is defined as the deviation of the sticky price output from its flexible-price counterpart, nominal risks are lower and real risks are higher.

We find that a time-varying inflation target with learning helps to better match the standard deviations of finance moments, thereby confirming findings in the literature. However, we also identify a negligible effect on the means of finance variables such as the mean of the nominal term premium. We argue that the introduction of price rigidity or higher price rigidity in the form of some real rigidity such as firm-specific fixed capital helps to generate a higher nominal term premium, thereby generating results closer to empirical findings.

We conduct welfare evaluation with the losses calculated in consumption equivalent terms of a particular Taylor rule relative to the deterministic steady-state. We find that the flexible price version yields lower welfare losses than the simple gap version as the output gap is smaller in flexible price gap case. The difference between the simple and flexible outgap models in terms of welfare is can be large in case of the high output gap coefficient.

Finally, we estimate two versions (simple and flexible price output gap) of the model extended with richer fiscal setup and more detailed labour market on US data 1961-2007 using the GMM and SMM. We find that the GMM and SMM parameter estimates are quite similar. The interest rate rule containing the flexible price version of the output gap stabilizes the economy more and hence, implies lower inflation risks and a lower variability of the nominal term premium on nominal bonds for given risk-aversion. Overall, we find that the simple gap version of the model seems to fit a set of macroeconomic moments better than the flexible price gap model. However, the mean of the nominal term premium is similar across the estimated models as the estimated risk-aversions and the standard deviations of the shocks are roughly identical.

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10 Appendix A–Welfare evaluation

As common in the literature (see, e.g., Schmitt-Grohe and Uribe (2007)), we calculate three measures of welfare to rank the various form of Taylor rules analyzed in the paper. We report the conditional and unconditional welfare expressed in consumption equivalents. The conditional welfare has been proposed by Schmitt-Grohe and Uribe (2007) to ensure that the economy begins from the same initial point under all possible polices. The third measure is based on the minimized value of the asymptotic loss function which ranks the Taylor type rules from the point of view of monetary policy authority.

Welfare based evaluation relies on the assumption that optimal monetary policy should minimize the distortions (frictions in the model) to achieve highest possible welfare. The welfare function can be recursively written,

$$V(Z_t, S_t; \sigma) = \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-L_t)^{1-\chi}}{1-\chi} + \beta \left[E_t V(Z_{t+1}, S_{t+1}; \sigma)^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \tag{14}$$

where S_t is a vector of state variables, Z_t vector of shocks and σ is the perturbation parameter. The difference between conditional and unconditional welfare lies in the form of expectations. For the conditional case we evaluate $E_t \left[V(Z_{t+1}; S_{t+1}; \sigma)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$ where we condition on the state of the economy at time t. Whereas $E \left[V(Z_{t+1}; S_{t+1}; \sigma)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$ can be used to calculate the unconditional welfare. We take a third-order approximation of the welfare function joint with the rest of the model to calculate welfare.

The welfare cost is reported in terms of consumption equivalents relative to the deterministic steady state. More specifically, we define consumption equivalents as the percentage compensation in the consumption process associated with the policy rule necessary to make the level of welfare under the steady state identical to that under the considered policy. Thus, a positive figure indicates that welfare is higher under the deterministic steady state than under the considered policy.

The third measure we calculate the minimized value of the asymptotic loss function following Svensson (2002) who argues that social welfare is too complex to serve as an operational objective for central banks. He claims that central banks should instead smooth inflation, output gap while minimizing the changes in their policy rule. In particular, he proposes to minimise the variance of inflation, output gap and the changes in the nominal interest rate as follows:

$$\min_{\phi_x, \phi_\pi} (\widetilde{\lambda_1} var(\pi) + var(x) + \widetilde{\lambda_2}(r_t - r_{t-1})) \tag{15}$$

subject to

$$r_t = \bar{r} + \phi_\pi(\pi_t - \bar{\pi}) + \phi_x(y_t - \bar{y}) \tag{16}$$

where π stands for inflation, y for output, r nominal interest rate, ϕ_y, ϕ_π are the weights on output and inflation in the policy rule, respectively. $\widetilde{\lambda_1}$ and $\widetilde{\lambda_2}$ are contanst weights. We follow Levin et al. (2008) and set $\widetilde{\lambda_1} = 16$ and $\widetilde{\lambda_2} = 1$.

We augment Tables 3-5 from the paper by the three welfare measures. We denote conditional and unconditional welfares as λ and λ^u, respectively. The third measure which is the minimised value of the assymptotic loss function is denoted as Loss. In particular, the tables 9, 11, 13 replicate the tables 3, 4, 5 in the main text of our paper for the models using the simple form of output gap (the difference between sticky price output and the steady-state output) and tables 10, 12, 14 for output gap in its flexible price form (the difference between sticky price and flexible price outputs).

As the sign of the conditional and unconditional welfare measures are the same we report only one of them (the conditional welfare measure) in the main text of our paper to keep the clarity of the exposition. In the following we provide an example for the interpretation of welfare measures: for instance in the model with RS rule households are missing 0.72% of their life time consumption to reach the welfare they would enjoyed in deterministic steady state and in the model from the first row of table 9 the welfare is lower by 0.91% of households life time consumption.

Table 9
Simple output gap and low coefficient on the output gap.

Case	Meaning	NTP	RTP	IRP	λ	λ^u	Loss
Case 1	Π_t	0.69	0.16	0.53	0.91	1.80	0.30
Case 2a	Π_{t+1}	0.73	0.15	0.58	0.97	2.02	0.24
Case 2b	$\bar{\Pi}_t$	0.50	0.06	0.44	0.84	1.45	0.14
Case 3a	1 w/ smoothing	0.52	0.10	0.43	0.87	1.37	0.21
Case 3b	2b w/ smoothing	0.42	0.04	0.38	0.84	1.21	0.11
Case 4	RS rule	0.31	0.04	0.26	0.72	0.75	0.01

Notes: The structure of this Table follows the structure of Table 3-5 in the main text of the paper, see more information there about the rules in each row (in short: the first row indicates that inflation is in contemporaneous form in the Taylor rule (Π_t); the second row contains inflation in forward-looking form (Π_{t+1}) ; the third row contains a 4-quarter moving average of inflation ($\bar{\Pi}_t$); smoothing in the fourth and fifth rows indicates interest rate smoothing in the Taylor rule; that last row contains the particular Taylor rule in Rudebusch and Swanson (2012)). In Table 3, all interest rate rules contain the definition of the simple output gap: i. e. the deviation of the sticky price output from its steady-state. NTP, RTP and IRP denote the nominal, real and inflation risk premium, respectively. IRP is calculated as NTP-RTP. λ and λ^u stand for conditional and un conditional welfare expressed in consumption equivalents, respectively. Loss stands for the minimised value of the asymptotic loss function in equation (15). In this table, there are no long-run inflation target risks.

Table 10
Low coefficient on the output gap and flexible price version of the output gap

Case	Meaning	NTP	RTP	IRP	λ	λ^u	Loss
Case 1	Π_t	0.45	0.20	0.26	0.71	0.78	0.02
Case 2a	Π_{t+1}	0.47	0.19	0.28	0.71	0.79	0.02
Case 2b	$\bar{\Pi}_t$	0.39	0.16	0.23	0.69	0.76	0.02
Case 3a	1 w/ smoothing	0.37	0.16	0.21	0.70	0.76	0.02
Case 3b	2b w/ smoothing	0.33	0.14	0.19	0.69	0.74	0.02
Case 4	RS rule	0.30	0.08	0.22	0.69	0.72	0.01

Notes: Low coefficient on the output gap (g_y = 0.07). Here output gap means the flexible price version of output gap, i.e., the difference between sticky price output and flexible price output. No long-run inflation target risks.

Table 11
High output gap coefficient and simple output gap

Case	Meaning	NTP	RTP	IRP	λ	λ^u	Loss
Case 1	Π_t	1.61	-0.01	1.61	3.60	5.52	136.91
Case 2a	Π_{t+1}	1.69	-0.02	1.71	4.00	6.22	179.62
Case 2b	$\bar{\Pi}_t$	1.18	-0.18	1.36	3.01	4.38	83.54
Case 3a	1 w/ smoothing	1.26	-0.14	1.40	3.12	4.60	92.54
Case 3b	2b w/ smoothing	0.98	-0.24	1.22	2.73	3.81	61.79
Case 4	RS rule	0.46	-0.09	0.55	0.99	1.06	0.04

Notes: Taylor rule specifications when the coefficient on the output gap is close to one: High output-gap coefficient (g_y = 0.93). The case for simple output gap. No long-run inflation target risks.

Table 12
High output gap coefficient and flexible output gap

Case	Meaning	NTP	RTP	IRP	λ	λ^u	Loss
Case 1	Π_t	0.27	0.22	0.05	0.65	0.67	0.01
Case 2a	Π_{t+1}	0.27	0.22	0.05	0.65	0.67	0.01
Case 2b	$\bar{\Pi}_t$	0.27	0.22	0.04	0.65	0.67	0.01
Case 3a	1 w/ smoothing	0.25	0.21	0.04	0.65	0.67	0.01
Case 3b	2b w/ smoothing	0.25	0.21	0.04	0.65	0.67	0.01
Case 4	RS rule	0.28	0.18	0.10	0.65	0.68	0.01

Notes: Taylor rule specifications when the coefficient on the output gap is close to one: High output-gap coefficient (g_y = 0.93). The case for flexible output gap. No long-run inflation target risks.

Table 13
High output gap coefficient, simple output gap and long-run inflation target risks

Case	Meaning	NTP	RTP	IRP	λ	λ^u	Loss
Case 1	Π_t	1.53	-0.00	1.53	8.66	31.45	4972.16
Case 2a	Π_{t+1}	1.61	-0.02	1.63	9.05	31.57	4744.18
Case 2b	$\bar{\Pi}_t$	1.05	-0.16	1.21	8.18	31.30	5515.91
Case 3a	1 w/ smoothing	1.14	-0.13	1.27	8.25	31.37	5438.26
Case 3b	2b w/ smoothing	0.82	-0.21	1.03	7.96	31.11	5864.69
Case 4	RS rule	0.46	-0.09	0.55	1.02	1.09	0.04

Notes: Taylor rule specifications when the coefficient on the output gap is close to one: High output gap coefficient and long-run inflation target risks. The case for simple output gap.

Table 14
High output gap coefficient, flexible price version of the output gap and long-run inflation target risks

Case	Meaning	NTP	RTP	IRP	λ	λ^u	Loss
Case 1	Π_t	0.26	0.22	0.04	0.95	1.24	0.12
Case 2a	Π_{t+1}	0.26	0.22	0.04	0.95	1.25	0.12
Case 2b	$\bar{\Pi}_t$	0.26	0.22	0.04	0.94	1.22	0.11
Case 3a	1 w/ smoothing	0.24	0.21	0.03	0.94	1.23	0.11
Case 3b	2b w/ smoothing	0.24	0.21	0.03	0.94	1.21	0.10
Case 4	RS rule	0.28	0.18	0.10	0.66	0.68	0.01

Notes: Taylor rule specifications when the coefficient on the output gap is close to one: High output gap coefficient and long-run inflation risks. The case for flexible output gap.

As a note, here, we also provide the derivations of consumption equivalents. Specifically, we compute the welfare cost of a particular monetary and fiscal regime relative to the steady state (or RS benchmark rule) as follows

$$V_0^r = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^r, n_t^r) \tag{17}$$

where c_t^r and n_t^r denote the contingent plans for consumption and hours under policy regime r. Similarly, define the welfare associated with policy regime in steady state as,

$$V_0^{ss} = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^{ss}, n_t^{ss}) \tag{18}$$

Let λ denote the welfare cost of adopting policy regime r instead of the reference policy regime which is the deterministic steady state.

We measure λ as the fraction of steady state consumption (regime ss) that a household would be willing to give up to be as well off in steady state as under regime r. Formally, we can define λ which equalises the welfare across regimes:

$$V_0^r = E_0 \sum_{t=0}^{\infty} \beta^t U((1-\lambda)\, c_t^{ss}, n_t^{ss}) \tag{19}$$

For the particular functional form of the utility joint with the Epstein Zin curvature, the value function for our model can be written as:

$$\frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-L_t)^{1-\chi}}{1-\chi} + \beta \left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} = \frac{((1-\lambda)\, C)^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-L)^{1-\chi}}{1-\chi} + \beta \left[E_t V^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \tag{20}$$

where V is given by in the steady-state as:

$$V = \frac{((1-\lambda)\, C)^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-L)^{1-\chi}}{1-\chi} + \beta \left[E_t V^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \tag{21}$$

which simplifies to the steady state relationship

$$V = \frac{((1-\lambda)\, C)^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-L)^{1-\chi}}{1-\chi} + \beta V \tag{22}$$

Plugging it back and expressing for λ gives way to:

$$V_t^r = \frac{1}{1-\beta} \left(\frac{((1-\lambda)\, C)^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-L)^{1-\chi}}{1-\chi} \right) \tag{23}$$

which can be also written as:

$$V_t^r (1-\beta) - \chi_0 \frac{(1-L)^{1-\chi}}{1-\chi} = \frac{((1-\lambda)\, C)^{1-\varphi}}{1-\varphi} \tag{24}$$

Some further algebraic manipulation leads us to express λ as:

$$\lambda = \left(1 - \left[V_t^r (1-\beta) - \chi_0 Z^{1-\varphi} \frac{(1-L)^{1-\chi}}{1-\chi} \right]^{\frac{1}{1-\varphi}} \frac{(1-\varphi)^{\frac{1}{1-\varphi}}}{C} \right) \tag{25}$$

which is the expression we used in our tables to calculate the conditional welfare cost.

11 Appendix B–Extended Model

To facilitate a better fit of the model in terms of real wage and hours growth, we introduce labor market frictions à la Blanchard and Gali (2007) into the model. In particular, we assume that there is wage bargaining between workers and firms and that the real wage can be well approximated by the following inertial process:[24]

$$w_t = \mu w_{t-1} + \mu(w_t^* + \bar{\omega}). \tag{26}$$

In equation (26), ω denotes the steady-state wedge[25] between the real wage and households' marginal rate of substitution, while μ refers to the sluggishness of wages in adjusting toward the frictionless real wage. Here, w_t denotes the reservation wage of workers and w_t^* the frictionless level of the wage that obtains in the absence of frictions in the labor market and is equal to the marginal rate of substitution between consumption and leisure (also called the intratemporal condition):

$$w_t^* = \varphi \hat{c}_t + \frac{\bar{L}}{1-\bar{L}} \chi \hat{l}_t + d\tau_t, \tag{27}$$

where $\hat{c}_t \equiv \log(C_t/\bar{C}), \hat{l}_t \equiv \log(L_t/\bar{L})$, and $d\tau_t \equiv \tau_t - \bar{\tau}$.
The marginal cost is defined—in log-linear terms—as the difference between the real wage and the marginal product of labor:

$$\begin{aligned} \widehat{mc}_t &\equiv \log(mc_t/\overline{mc}) = \widehat{w}_t - \widehat{mpl}_t \\ &= \widehat{w}_t - (\hat{a}_t + (\eta - 1)\hat{l}_t). \end{aligned} \tag{28}$$

In equation (28), $\hat{a}_t \equiv \log(A_t/\bar{A}), \widehat{w}_t = \log(W_t^r/\bar{W}^r)$, $W_t^r \equiv W_t/P_t$ and $\widehat{mpl}_t \equiv \log(MPL_t/\overline{MPL})$ denote the log-deviations of the technology shock, the real wage and the marginal product of labor from the corresponding steady-states values (captured by an upper bar), respectively. The first row contains the definition of the real marginal cost in log-linear form. The second row contains the marginal product of labor based on the Cobb-Douglas functional form.
Using equations (28) and (27), we can rewrite equation (26) into:

$$\widehat{mc}_t - \widehat{mpl}_t = \mu(\widehat{mc}_{t-1} - \widehat{mpl}_{t-1}) + (1-\mu)\left(\varphi \hat{c}_t + \frac{\bar{L}}{1-\bar{L}} \chi \hat{l}_t + d\tau_t + \bar{\omega}\right) \tag{29}$$

As equation (29) shows, higher taxes imply a gradual increase in marginal costs due to sluggishness caused by real wage rigidity. Note that real wage rigidity helps to raise the nominal term premium by 10-25 basis points on average due to the fact that labor which works as a vehicle to insure against bad shocks can be adjusted slower in the presence of real wage rigidity.[26]

Regarding the extension on the fiscal side we assume that the government's budget is balanced each period by labor income tax revenue and the steady-state government debt is positive. Hence, the government budget constraint can be written as:

$$g_t + \frac{B}{Y}\left(\frac{R_{t-1}}{\Pi_t} - 1\right) = \tau_t w_t L_t. \tag{30}$$

In the previous equation $\frac{B}{Y}$ denotes the steady-state government debt-to-GDP ratio.

[24] Note that that model extension here is written in linear terms for illustration only. The model is in fact is approximated to the third-order.

[25] The choice of $\bar{\omega}$, which equals the log of the so-called wage markup, does not influence our results.

[26] Li and Palomino (2014) highlight the importance of nominal wage rigidity in matching macro and finance moments when the source of risks are permanent productivity shocks. In contrast the uncertainty in our model can be traced back predominantly to temporary technology and government spending shocks.

12 Appendix C–Data sources

To construct the following time series, we follow the procedures in Christoffel et al. (2013) and Leeper et al. (2010):

PY: Gross Domestic Product. Bureau of Economic Analysis (BEA). Nipa Table 1.1.5, line 1.

P: GDP deflator personal consumption expenditures. Source: BEA, Nipa Table 1.1.4, line 2.

C: Private Consumption. Source: BEA, Nipa Table 1.1.6, line 2.

L: hours, measure of the labour input. This is computed as $L = H \times (1 - U/100)$, where H and U are the average over monthly series of hours and unemployment, respectively. Source: BLS, series LNU02033120 for hours and LNS14000000 for unemployment.

INT: Net Interest Payments of Federal Government Debt. Source: BEA, Nipa Table 3.2 (line 29-line13).

G: Government consumption is computed as current consumption expenditures (line 21)+gross government investment (line 42)+net purchases of non-produced assets (line 44)-consumption of fixed capital (line 45). Source: BEA, Nipa Table 3.2

W: Wage and Salary Disbursement. BEA. Series ID A576RC1.

WL: labour income tax base. Source: Nipa Table 1.12 (line 3).

τ: average effective labour income tax rate as in Jones (2002) and Leeper et al. (2010). We follow the procedure in the appendix of Leeper et al. (2010) to construct τ_t.

B/Y: government-debt-to-GDP ratio. St. Louis Fed Database.

NTP: nominal term premium is obtained from the database of Adrian et al. (2013).

MNB Working Papers 2021/2

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Budapest, June 2021

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