CEIOPS- FS-13/07

QIS3
Technical Specifications

Annexes

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ANNEX A

Simplified liability adequacy test for non-life insurance

A.1 This method is intended to identify the main cases where expected expenses corresponding to a given non-life insurance portfolio are higher than their corresponding unearned premium provisions.

A.2 Let's assume for simplification that the insurer issued three annual premiums on April 1st, July 1st and October 1st 2006. Let's assume also that these three premiums correspond to renewals.

A.3 Starting from premiums and the corresponding claims, the following table determines the pre-claims provision.

	Premiums	**Claims and expenses before renewal date**	**Claims and expenses between renewal date and next 31/12**
2006 April 1st	1.200	Not considered	1.000
2006 July 1st	2.400	Not considered	0
2006 October 1st	4.800	Not considered	3.000
Earned premium ----->	1200*9/12 + 2.400*6/12 + 4.800*3/12 = 3.300	Total claims after renewal date -->	4.000
Liability adequacy test	3.300 – 4.000 = - 700		
Unearned premium	1200*3/12 + 2.400*6/12 + 4.800*9/12 = 5.100		
Pre-claims technical provision	5.100 + 5.100 * (700 / 3.300) = 6.182		

Annex B

Principles on the recognition of risk mitigation tools in the context of a standard formula calculation of the SCR

B.1 The principles laid out below refer to financial risk mitigation and are inspired on consistent practices and regulations of other financial sectors. They may be used to complement the advice on risk mitigation instruments that CEIOPS has given in its answer to CfA12, and are not intended to be applied to the field of reinsurance. A financial risk mitigation technique must meet these principles for the recognition in the standard SCR of its effect on an insurance undertaking's risk profile. By its own nature, principles should be enduring in time and permanent.

B.2 CEIOPS has not yet reached a final position on this issue, and participants are invited to comment on the appropriateness of the set of principles described in the annex in the context of a standard formula calculation of the SCR.

B.3 In cases where participants apply risk mitigation instruments for the calculation of the QIS3 standard formula SCR which do not fulfil the principles included in the annex, and where such mitigating instruments have a significant impact on the SCR, it is recommended that the participant indicates which of the principles were violated, and gives an estimation of the impact of the instruments out of the scope of the annex on the SCR estimate.

Principle 1: Economic effect over legal form

B.4 For standard SCR purposes, financial risk mitigating techniques that have a material economic impact on an insurance undertaking's risk profile, should be recognised and treated equally, regardless of their legal form or accounting treatment, provided that their economic or legal features do not oppose to the principles and rules required for such recognition.

B.5 New risks acquired as a by-product of financial risk mitigating techniques should also be recognised for standard SCR purposes.

Principle 2: Legal certainty, effectiveness and enforceability

B.6 The techniques used to provide the financial risk mitigation together with the action and steps taken and procedures and policies implemented by the insurance undertaking shall be such as to result in risk mitigation arrangements which are legally effective and enforceable in all relevant jurisdictions.

B.7 The insurance undertaking shall take all appropriate steps, for example a legal review, to ensure and confirm the effectiveness and continuing enforceability of the financial risk mitigation arrangement and to address related risks.

B.8 Undocumented or deficiently documented financial risk mitigation techniques should not be considered, even on a partially sufficient basis, for standard SCR purposes.

Principle 3: Liquidity, ascertainability and stability of value

B.9 To be eligible for recognition, the financial risk mitigation instruments relied upon shall have a value over time sufficiently reliable and stable to provide appropriate certainty as to the risk mitigation achieved.

B.10 Regarding liquidity, QIS3 specifications do not contain any concrete requirement, but only the following two general statements:

a) The insurer should have written guidance regarding liquidity requirements that financial risk mitigation instruments should meet, according the objectives of the own insurer's risk management policy,

b) Financial risk mitigation instruments considered to reduce the SCR have to meet the liquidity requirements established by the own entity.

B.11 CEIOPS welcomes comments from external stakeholders on liquidity requirements, if any, that may be sensible to impose, especially regarding financial risk mitigation instruments with a long term.

B.12 The standard SCR calculation should recognise financial risk mitigation techniques in such a way that there is no double counting of mitigating effects and no higher capital requirement is obtained than if there were no recognition in the standard SCR of such mitigation techniques.[1]

Principle 4: Credit quality of the provider of the risk mitigation instrument

B.13 Providers of financial risk mitigation should have an adequate credit quality to guarantee with almost certainty that the insurer will receive the protection in the cases specified by the contracting parties. Credit quality should be assessed using objective techniques according generally accepted practices.

B.14 As a general rule, when the insurer applies the standard calculation for a certain risk module, only financial protection provided by entities rated BBB or better will be considered in the assessment of SCR. In the event of the default, insolvency or bankruptcy of the provider of the financial risk mitigation instrument – or other credit event set out in the

[1] This is intended to mean that where the risk mitigation technique actually reduces risk, the SCR should not be increased. On the other hand, where the risk mitigation technique does not work as intended and actually increases risk, then, of course, the SCR may be increased.

transaction document – the financial risk mitigation instrument should be capable of liquidation in a timely manner or retention. The degree of correlation between the value of the instruments relied upon for risk mitigation and the credit quality of their provider shall not be undue, i.e. is material positive.

B.15 When the insurer applies an approved internal model that

a) explicitly identifies the risks derived from using a financial protection provider rated less than BBB,

b) and quantifies the appropriate capital charges associated to those risks,

the protection of such counterparty may be admitted to reduce the SCR corresponding the risk internally modelled.

Principle 5: Direct, explicit, irrevocable and unconditional features

B.16 Financial mitigating instruments only could reduce the capital requirements if :

a) They provide the insurer a direct claim on the protection provider (direct feature),

b) They contain explicitly reference to specific exposures or a pool of exposures, so that the extent of the cover is clearly defined and incontrovertible (explicit feature),

c) They do not contain any clause that would allow the protection provider unilaterally to cancel the cover or that would increase the effective cost of cover as a result of certain developments in the hedged exposure (irrevocable feature),

d) They do not contain any clause outside the direct control of the insurer that could prevent the protection provider from being obliged to pay out in a timely manner when it comes due according contractual clauses (unconditional feature).

Special features regarding credit derivatives

B.17 Reduction of standard SCR based on the mitigation of credit exposures using credit derivatives will be allowed only if the insurer meets similar requirements as other financial activities for the same mitigating techniques. The application of this requirement will be made according a principle of proportionality.

B.18 In order for a credit derivative contract to be recognised, the credit events specified by the contracting parties must at a minimum cover:

- failure to pay the amounts due under terms of the underlying obligation that are in effect at the time of such failure (with a grace period that is closely in line with the grace period in the underlying obligation);

- bankruptcy, insolvency or inability of the obligor to pay its debts, or its failure or admission in writing of its inability generally to pay its debts as they become due, and analogous events;

- and restructuring of the underlying obligation. Since the definition of 'restructuring' is not fully harmonised at international level, for QIS3 purposes the precise identification of this event will be left to the own insurer's discretion, according its risk management policy.

Collateral

B.19 A collateralised transaction is one in which insurers have a credit exposure or potential credit exposure and it is hedged in whole or in part by collateral posted by a counterparty or by a third party on behalf of the counterparty.

B.20 In addition to the general requirements for legal certainty, the legal mechanism by which collateral is pledged or transferred must ensure that the insurer has the right to liquidate or take legal possession of it, in a timely manner, in case of any event of the counterparty set out in the transaction documentation (and, where applicable, of the custodian holding the collateral).

B.21 Insurers must have clear and robust procedures for the timely liquidation of collateral to ensure that any legal conditions required for declaring the default of the counterparty and liquidating the collateral are observed, and that collateral can be liquidated promptly.

B.22 Unless it becomes impossible according market conditions, admissible collateral for standard SCR purposes must protect the insurer against the same events listed in this paper for credit derivatives.

ANNEX C

Supplementary guidance note on life technical provisions

Illustration of some possible less advanced approaches that could be applied in the estimation of the best estimate liability

C.1 Summary

C.1.1 It should be emphasised that participants in QIS3 are asked to apply the general valuation specifications on a best effort basis. Participants are thus allowed to take part on an approximate basis and focus on material issues if that is the best what is achievable in the time available to perform the valuations.

C.1.2 The following is a summary of some of the possible less advanced approaches illustrated in this guidance note that could be applied in the estimation of the best estimate liability:

- Grouping of contracts
    - o Choose suitable model points within a specimen policy instead of modelling each contract in the portfolio separately.
- Biometric assumptions
    - o Neglect the reflection of the so-called trend forecast.
    - o Apply a period instead of a cohort approach for mortality.
    - o Use assumptions currently available in the valuation of technical provisions appropriately scaled to obtain an approximation for the best estimate.
    - o Use a deterministic approach and neglect any possible need for stochastic simulation.
    - o Assume independence from any other variable.
- Surrender option
    - o Assume that surrenders occurs independent of financial and biometric risks.

  - Assume that surrenders occurs independent of firm specific information.

  - Model the surrender as a hazard process either with a non-constant or constant intensity.

- Financial options and guarantees

  - Approximate guarantees and options by assuming a Black-Scholes type of environment.

- Investment guarantees

  - Assume non-path dependency.

  - Focus on intrinsic values.

  - Apply formulaic simplified approaches for the time values if they are considered to be material.

- Other options and guarantees

  - Focus on material other options and guarantees.

  - Approximate for instance by grouping investment, mortality and expense guarantees into one single investment guarantee.

  - In the absence of well a defined valuation approach use subjective ad hoc approaches if the options and guarantees are considered to be of material importance.

- Distribution of extra benefits

  - Assume non-path dependency.

  - Assume a constant distribution ratio of extra benefits reflecting past practises. Apply the distribution ratio to the overall valuation to determine the amount of extra benefits and the time values of guarantees.

  - Alternatively, approximate the amount of available extra benefits for distribution to policyholders as the difference of liabilities currently held and the present value of expected future guaranteed benefits adjusted with appropriate considerations to future expenses needed to service the insurance contracts.

- Expenses and other charges

  - Use information from current expense loadings, future projected expense loadings and past expense analysis.

- Assume other charges to be a constant reduction of extra benefits or a constant charge from the policy fund.

- Other issues
  - Chose a projecting period equal to one year.
  - Assume cash-flows to occur at the end or in the middle of the time intervals.
  - Assume that future premiums are paid independently of the financial market and firm specific information or alternatively neglect the premiums.

C.2 Introduction

C.2.1 The objective of this guidance note is to illustrate some possible less advanced approaches that could be applied in the valuation of technical provision and especially in the valuation of the best estimate. Since this is an area where the best practise is still under development this guidance note should not in any way be considered to be fully exhaustive. It should also be noted that the illustrations might not be directly applicable for all firms in all jurisdictions and therefore company specific adjustments might be needed to derive solutions that can be implemented in practice. However in general regarding any simplifications a firm may take from a more comprehensive approach, care should be taken by the firm to understand its true exposure to different risks, and to disclose their nature and the simplifications assumed. Without such real understanding by the firm and without any documentation[2], it will be difficult to have a sufficient confidence on the valuation. Moreover, for certain areas it may be useful to adopt an iterative development process where approximations are gradually improved.

C.3 Grouping of contracts

C.3.1 In general the valuation of the best estimate liability should be based on policy-by-policy data. However reasonable actuarial methods and approximations may be used and in particular the projection of future cash flows based on suitable specimen policies can be permitted.

C.3.2 It should be noted that two policies can in principle be grouped together if they have in sensible way the same issue age, issue date, sex, mix of underlying funds, investment guarantee and nature of embedded options (in-the-money or out-of-the money) etc. Choosing suitable model points within a specimen policy instead of modelling each contract in the portfolio separately could provide a reasonable first approximation.

[2] In QIS3 no strict documentation is required, but with respect of any future review process it would become a necessity.

C.3.3 However, appropriate attention should be given to the possibility that the grouping of policies into a specimen policy could materially misrepresent the underlying exposure and could significantly misstate the costs.

C.4 Biometric assumptions

C.4.1 In general, a best estimate assumption for a biometric factor consists of two main parts: (1) the currently observed experience and (2) the expected changes in this level of risk in the future. The first reflects the historical average up to the valuation date and the second one reflects the so-called trend forecast. For life firms mortality is probable one of the most significant risk factor and therefore this risk factor will be elaborated further.

C.4.2 Cohorts and periods are two different approaches to observe the evolution of time when modelling mortality. A cohort approach is composed of individuals who are born in the same year (decade or some other grouping of time) whereas a period approach is a time interval where mortality is observed for a group of people on conditions prevailing during that period and factors that simultaneously affect the whole population. Mainly due to statistical reasons the period approach have traditionally been used, but in recent years the cohort approach has gained significant ground. Traditional analyses of mortality however fail to describe differences in mortality between generations. In contrast a cohort approach relates to lifelong exposure to mortality risk factors shared by a whole generation and thus allows more flexible forecasting possibilities.

C.4.3 Mathematically this can be defined (here ignoring the grouping into sex) by making references to probability distributions or more simply to the expected force (intensity) of mortality. The expected force of mortality within a cohort approach would typically be expressed as $\mu_c(x,\tau)$, which defines the expected force of mortality at age x for the cohort born at time τ. Similarly for a period approach it can be expressed as $\mu_p(x,t)$, which defines the expected force of mortality at age x at time t. The valuation can be simplified by assuming that the link between the cohort and the period intensity is given by the relation

$$\mu_c(x,\tau) = \mu_p(x,\tau + x) = \mu_p(x,t) = \mu(x,t),$$

that is by assuming that the age distribution is identical from cohort to cohort. Given this simplification the next questions arises from the inclusion of trends and their approximations. A straight-forward approach can be based on the current mortality table in use, which is adjusted for example by a suitable multiplier function.

C.4.4 Let $\mu_p(x,t_0)$ denote an expected standard period life table that reflects the historical information up until time t_0. Assume further that a monotonically decreasing reduction factor $R(x,t)$ with respect of t is defined that takes into account the expected changes in the future (trends) so that

$$\mu_p(x,t) = \mu_p(x,t_0) \cdot R(x,t)$$

for $t \geq t_0$ and where $R(x,t_0) = 1$ for all ages x and $0 < R(x,t) \leq 1$ for all $x, t > 0$. The challenging parts are clearly the construction of a reliable life table and the modelling of an adequate reduction factor, which can be problematic for many firms. Here industry-wide and other public data and forecasts should provide useful benchmarks whenever available (for example a similar relative change in mortality during the forecast period as in the public forecast might be considered).

C.4.5 Accordingly to the in-force EU-directives the force of mortality currently in use within the valuation of technical provisions should be prudent. Hence a possible approximation would be to assume that

$$\mu_p(x,t) = \alpha(x,t) \cdot \mu_{current}(x,t),$$

where $\alpha(x,t)$ represent an age and time dependent scaling factor. If the data available is extremely thin a last possible approximation would be to assume an age and time independent scaling factor, that is

$$\mu_p(x,t) = \alpha \cdot \mu_{current}(x,t).$$

C.4.6 The scaling factor should aim for a best estimate and be prudently chosen taken into account the uncertainty of the estimate. However, margins for adverse circumstances should not deliberately be included.

C.4.7 Traditionally reserves have been valued using prudent and deterministic force of mortality assumptions. By modelling the force of mortality in stochastic ($\tilde{\mu}_p(x,t)$ or $\tilde{\mu}_c(x,t)$)[3] manner the uncertainty related to the future development can however be more appropriately captured and quantified. For large portfolios it is commonly assumed that the law of large numbers causes the variation to be rather narrowly spread around the mean, which would indicate that a deterministic approach would be acceptable (except possibly for the trend forecast, for which the error is not diversifiable within a line of business). However, when the liability to be valued contains for instance embedded options that depend on the mortality (creating unsymmetrical profiles) a stochastic approach has its clear advantages. Stochastic mortality models are typically based on statistical models such as time series and multivariate analysis and generalised linear models, (c.f. the Lee-Carter approach), or modelling frameworks analogous to short-rates and forward-rates etc.

C.4.8 For most firms the requirement of stochastic mortality modelling is rather challenging. A possible fall-back approach for mortality related options and guarantees could be to include some part of the liability exposure for instance in the valuation of the investment-related part (similar options and guarantees).

[3] Tilde denotes a stochastic variable.

C.5 Surrender option

C.5.1 The surrender option (sometimes also called withdrawal option) embedded in many life insurance contracts gives the policyholder the right to terminate the contract before maturity and to receive a cash amount commonly called the surrender value. The cash amount is commonly pre-determined accordingly to some principles, but on future dates not necessary known in advance (stochastic).

C.5.2 The surrender option is a very important element for a firm and it should be taken into account in the valuation of technical provisions. It should be noted that a surrender option could also have significant financial effect for instance on uncharged expenses and uncharged costs for options and guarantees.

C.5.3 Inspired by financial theory on credit risk valuation two broad approaches are usually distinguished in the literature for modelling the surrender option, which are so-called (1) structural models and (2) intensity models. The first approach tries to encompass rational behaviour whereas the second approach tries to encompass more irrational behaviour in the modelling.

C.5.4 Structural models derive the price of the surrender option by modelling the realistic value of the contract relative to the surrender value as an American option. These options are theoretically modelled using the theory of stochastic processes and their optimal stopping time. Using the notions of this theory an answer to the valuation problem can be described as an optimal stopping time with respect to the filtration generated by the prices of the financial assets.

C.5.5 Given a complete probability space $(\Omega, \mathscr{F}, P)$ and $\omega \in \Omega$ it can be illustrated as

$$\tilde{\tau} = \inf\{t \mid \tilde{R}(t,\omega) \geq \tilde{V}(t,\omega)\},$$

where $\tilde{\tau}$ denotes the random time of the surrender, $\tilde{V}(t,\cdot)$ denotes the random realistic value of the insurance contract at time t if it has not been surrendered and $\tilde{R}(t,\cdot)$ denotes the random amount the policyholder receives upon surrender of the contract at time t. Thus it is optimal for the policyholder to surrender when the realistic value of the contract is inferior or equal to the amount received by immediately surrendering the contract. In practice American options are commonly priced recursively using numerical methods such as binomial trees.

C.5.6 The pros of this approach are that the surrender time is endogenously determined and the surrender time emerges from the characteristics of the insurance contract and depends intrinsically on the evolution of the financial market and the valuation methods used therein (option pricing).

C.5.7 The cons of this approach are the following:

- It assumes that the policyholders receive and are able to process all the information required to make the calculations in real time.

- Policyholders are assumed to take their decisions on exactly the same set of information and all to act in the same way, which is in conflict with a portfolio approach typical in insurance business.

- It does not allow for any asymmetry of information between the policyholders and the firm.

- There is never any unexpected surrender due to policyholders' behaviour and therefore there is no room for idiosyncratic information that could trigger the decisions of surrender. It might therefore give an impression that the surrender risk can always be perfectly hedged away and that the surrenders are not a risk for the firm.

C.5.8 The structural approach thus set the contract into a contingent claims framework characterized by perfectly competitive, complete, frictionless, transparent and arbitrage-free market, driven by rational investors all sharing the same information. According to this approach the surrender is not at all independent of financial elements, since it is a consequence of a rational choice.

C.5.9 One could argue in favour for the structural approach that the market value of a right, such as the surrender option, should theoretically be independent of the behaviour of the owner and therefore even if the policyholder would very likely surrender the policy by exogenous reasons the correct approach to follow would still be the structural approach. In other words, the policyholder has the right to act optimally when taking the surrender decision and hence for prudential reasons no discount should be given for anticipated non-optimal behaviour. In fact, even if such discounts were allowed, the firm could not subsequently forbid the policyholder to act optimally. One could go further and ask: Are all traded American options always rationally exercised? And in any case the professional traders in option markets do not necessarily represent a typical life insurance policyholder.

C.5.10 The intensity approach observes that the "value" of the contract from the policyholder's perspective depends also on own information and own risk aversion (utility). It is thus a subjective value and a subjective value is different from a policyholder to another and is mostly unknown for the firm. It should also be noted that a rather common practise is that the information about the realistic value of a contract that needs to be compared with the surrender value, is not made public and kept as internal information by the firm.

C.5.11 On could argue in favour for the intensity approach that under most circumstances the structural (optimal stopping time) approach does not realistically model a policyholder's surrender behaviour and the existence of an asymmetry of information. The intensity model approach is not defined as a stopping time model where one can tell from the observation of the financial market if a policyholder has already surrendered or not and is thus more suitable for modelling realistic surrender behaviour. This approach can be theoretically described as follows.

C.5.12 Let $\tilde{\tau}$ be a non-negative random variable defined on $(\Omega, \mathscr{F}, P)$ with a filtration $\boldsymbol{F} = (\mathscr{F}_t)_{0 \le t \le T}$. Assume that $P(\tilde{\tau} = 0) = 0$ and that $P(\tilde{\tau} > t) > 0$ for all $t \in \boldsymbol{R}_+$. Set $F_t = P(\tilde{\tau} \le t \mid \mathrm{F}_t)$ for $t \in \boldsymbol{R}_+$ and postulate that $F_t < 1$ for all $t \in \boldsymbol{R}_+$. The $\boldsymbol{F}$-hazard process of $\tilde{\tau}$ under P, denoted by Γ, is defined by

$$F_t = 1 - e^{-\Gamma_t}$$

C.5.13 Note that a $\boldsymbol{F}$-stopping can not create a $\boldsymbol{F}$-hazard process characterisation. Thus

- the surrender decision depends necessarily on elements outside the information $\boldsymbol{F}$ shared by the insurer and the policyholder;
- these elements are idiosyncratic information or events and can be different from a policyholder to another;
- the surrender decision is also influenced by firm specific product structures;
- there is hence an implicit asymmetry of information between the policyholder and the insurer;
- due to asymmetric information the surrender decision is to some extent unpredictable for the firm. The probability to surrender is therefore stochastic and could be modelled to partially depend on the evolution of the financial market, which is to be reflected in Γ_t.

C.5.14 The assumptions made on $\tilde{\tau}$ are rather general and allows many ways to construct it in practice. One could for instance relate Γ_t to different stochastic factors driving the hazard process dynamically and keep the dependence on $\boldsymbol{F}$. The approaches could consist of multifactor functions that vary with realistic value, surrender value, age, policy duration, time to maturity, interest rates, market volatilities and other economic factors of importance. The dependence on the financial market would however result in rather comprehensive modelling more suitable for a stochastic simulation approach rather than a simple closed-form solution.

C.5.15 One can also think of $\tilde{\tau}$ as an Poisson process $\tilde{N}_t$ with hazard function Λ_t and independent of $\boldsymbol{F}$. Thus

$$F_t = 1 - P(\tilde{\tau} > t \mid \mathrm{F}_t) = 1 - P(\tilde{\tau} > t) = 1 - P(\tilde{N}_t = 0) = 1 - e^{-\Lambda_t}.$$

C.5.16 If F_t is differentiable, we can find the intensity γ_t of the Poisson process, which is a positive function, given by

$$\Gamma_t = \Lambda_t = \int_0^t \gamma_u du \ .$$

C.5.17 One possible simplification would be to model the rate at which policyholders surrender contracts could be to specify the intensity γ_t as a Poisson process and thus assume that surrenders occurs independent of the financial market. One choice is

$$\gamma_t = a - \frac{b}{t+c} \ ,$$

where $a \geq 0$, $b \geq 0$, $c > 0$ and $a > b / c$. Thus

$$F_t = 1 - e^{-a \cdot t} \left(\frac{t}{c} + 1 \right)^b$$

C.5.18 Perhaps the simplest approach would be to assume a constant intensity $\gamma_t = a$ over time. Then

$$F_t = 1 - e^{-a \cdot t}$$

C.5.19 It should be noted that the hazard process in the last two examples are deterministic functions and implies independence between the surrender time and the evolution of economic factors, which is obviously not a realistic assumption since policyholder behaviour is not static and is expected to vary as a result of a changing economic environment. Nevertheless, it is a very practical assumption which however requires special care and prudence needs to be exercised when estimating the parameters. The surrender decision is hence considered to be an exogenous cause of termination the contract and out of control of the firm.

C.5.20 Other possible surrender models where the surrender rate SR_t for a policy at time t (commonly with respect of some interval) also depends on economic variables are the following:

- Lemay's model: $SR_t = a \cdot \alpha_t + b \cdot \dfrac{FV_t}{GV_t}$
- Arctangent model: $SR_t = a + b \cdot \arctan(m\Delta_t - n)$;
- Parabolic model: $SR_t = a + b \cdot sign(\Delta_t) \cdot \Delta_t^2$;
- Modified parabolic model: $SR_t = a + b \cdot sign(\Delta_t) \cdot \Delta_t \cdot k + c^{(CR_{t-1} - CR_t)^j}$
- Exponential model: $SR_t = a + b \cdot e^{m \cdot \frac{CR_t}{MR_t}}$

- New York State Law 126:

$$SR_t = a + b \cdot sign(\Delta_t) \cdot \Delta_t \cdot k - c \cdot \left(\frac{FV_t - CSV_t}{FV_t} \right)$$

C.5.21 where *a, b, c, m, n, j, k* are coefficients, α denotes underlying (possible time dependent) base lapse rate, FV denotes the fund (account) value of the policy, GV denotes the guaranteed value of the policy, Δ equals reference market rate less crediting rate less surrender charges, CR denotes the crediting rate, MR denotes the reference market rate, CSV denotes the cash surrender value and sign() equals 1 if ( ) is positive and –1 if ( ) is negative. The common structure for these models is that the surrender rate is divided into two main parts consisting of a base rate reflecting irrational behaviour and a rate that depends on some economic factors reflecting rational behaviour.

C.5.22 So, one could obviously assume that mortality is independent of the financial market. A highly questionable but very practical assumption is to assume stochastic independence between surrenders and the financial markets and finally in order to simplify the valuation slightly more one could also assume stochastic independence between surrenders and mortality.

C.5.23 However, the surrender options and the minimum guarantees in with-profits contracts are clearly dependent. Policyholders will lose the guarantee if the surrender option is exercised. Furthermore, management actions (changes in asset allocation, distribution of extra benefits etc.) will also have a significant impact on the surrender option that cannot easily be captured in a closed formula.

C.5.24 It should also be noted that surrender options are usually assessed from a firm perspective on homogeneous groups rather than from a single policyholder perspective.

C.5.25 Moreover, firms' current IT-infrastructures could also cause additional challenges. Some firms may not have included the functionality to calculate the surrender values for all policies within their current valuation systems. Other separate systems may currently be in place for some contracts were the surrender values are calculated on a one-off basis, i.e. upon request by the policyholder.

C.5.26 Even after a model has been selected there is great challenge to estimate the parameters. One should not neglect that policyholder behaviour may change over time and that the currently observed surrender pattern could be a poor prediction of future behaviour. To what extent do policyholders become more sophisticated and markets more efficient as time passes? How would policyholder react to a bear market condition in financial markets given that we have recently observed a bull run and the opposite? How will policyholders react to a declining solvency position by the firm? The lack of credible data will indeed require a lot of judgements and the made assumptions should therefore be continuously monitored and changed if the emerged experience so would indicate. In addition alternative scenarios could be applied in these calculations.

C.5.27 It should be noted that many other options embedded in life insurance contracts have a nature similar to the surrender option and could therefore possible be modelled in a similar way. This includes for instance fund switching options (switching for instance between a traditional with-profit contract and a unit-linked contract), reset options (allows for instance to “lock in” investment gains) etc.

C.6 Financial options and guarantees

C.6.1 Life insurance contracts are commonly rather complex. Besides of having classical pure insurance elements they usually also have either explicitly or implicitly built in different kinds of financial options and guarantees. Perhaps most common are various kinds of financial guarantees.

C.6.2 The benefits of a with-profits contract for instance are linked to a reference fund that is influenced by the firm’s investment strategy. The insurance benefits of such a contract usually consist of at least two parts that are (1) guaranteed benefits and (2) variable periodic extra benefits (usually annual) that is based on the profits the firm has been able to generate from the policy fund and that is often added to the guaranteed benefits (reversionary extra benefit) and/or a variable terminal extra benefit that is not guaranteed until maturity and even then based on the final profits the firm has been able to generate from the policy fund.

C.6.3 The reversionary extra benefits in a with-profits contract can be interpreted as a series of consecutive forward start options (where the first starts immediately, the second when the first expires etc.) that periodically locks in profits. Due to discretion exercised by the management and the general strategies to run the business the link between the benefits, underlying assets and the values of the options is complex. It should be noted that the options are usually also influenced by legislative restrictions.

C.6.4 In contrast to unit linked products, the underlying asset portfolio for a with-profits fund is internally managed by the firm and not directly traded on the financial markets. Thus, hedging the financial options and guarantees with the underlying asset portfolio together with a risk-free asset causes additional valuation difficulties.

C.6.5 Financial options and guarantees can generally be valued accordingly to two main techniques that are:

1) If the factor is hedgeable on deep, liquid and transparent market, use observed market prices;

2) If the factor is non-hedgeable, mark-to-model i.e.:

    a) use stochastic simulation techniques (Monte Carlo or appropriate numerical partial differential equation approaches) to value the financial options and guarantees by considering a range of future stochastically varying economic conditions (interest rates or other underlyings), calibrated to a market consistent asset model and then calculating the average present value of the costs related to the options and guarantees to be valued. The connection to market consistent prices and arbitrage-free valuation is achieved by ensuring that the asset model reproduces observed market prices for some representative assets;

b) use a deterministic approach, where a series of deterministic projections of the values of the underlying assets are made and where each deterministic projection corresponds to a possible economic scenario together with the associated probability of occurrence. The costs of the financial options and guarantees equal the average costs generated by the probability weighted deterministic projections of the assets. The connection to market consistent prices and arbitrage-free valuation is achieved by ensuring that the probability weighted deterministic scenarios reproduced observed market prices for some representative assets;

c) use closed form estimate derived from an arbitrage-free model (e.g. Black-Scholes formula) with parameters calibrated to market prices of similar options.

C.6.6 It should be noted that the complexity of financial options and guarantees and other features (such as management actions for instance) in most insurance contracts commonly creates non-hedgeable risk factors. Hence the direct use of observable market prices is rather limited.

C.6.7 By performing stochastic projections accordingly to stochastic simulation or probability weighted deterministic projections of the cash-flows a distribution of the costs to meet the obligations from financial options and guarantees is obtained. However, Stochastic simulation is the preferred technique as it can deal appropriately with very complicated liability structure such as for-instance path-dependent behaviour. Its disadvantage is the need for sophisticated modelling, which rather often creates difficulties for less advanced firms.

C.6.8 For many firms closed form approaches are generally more practical. However, they are only suitable in special circumstances. Often various simplifying assumptions are made, such as the existence of complete financial markets, the stochastic dynamics of the underlying assets follow a geometric Brownian motion, use of dynamic hedging etc. which may distort the results.

C.6.9 By using the Black-Scholes option pricing formula one implicitly assumes the following (not a complete list):

- The existence of a risk-free and a risky (underlying reference portfolio) asset that can be traded continuously;
- The trading is frictionless (buying and selling price is the same);
- The volume of the trading does not affect the price and there are no transaction costs;
- Borrowing and lending can be made at the risk-free interest rate;
- At any time a certain percentage of the total value of the portfolio is invested in the risky asset and the rest in the risk-free asset;
- The allocation between the risk-free and the risky asset can be negative or exceed 100 percent (no restriction on short sales or borrowing);
- The portfolio of the risk-free and risky asset is self-financing (no external funds are added to or subtracted from the portfolio – rebalancing does not require any more or less cash);
- Dynamic replication – the portfolio is rebalanced continuously as to be risk-free (no risk premium is involved in the process – the whole package is constructed to be risk-free);
- The assets are infinitely divisible;
- The price process for the risky asset follows a geometric Brownian motion;
- Time independent volatility;
- Constant risk-free interest rate;
- Price of the option is the cost of setting up the replicating portfolio.

C.6.10 Whenever the market is complete a unique price exists, which explains the assumptions behind the Black-Scholes and many other option pricing formulas. However the conditions required to produce market completeness are quite stringent and within a real world these are never completely fulfilled. In particular such long-dated options that would allow the full term of life and pension insurance contracts to be hedged are not very commonly traded.

C.6.11 By applying the Black-Scholes option pricing formula to a life insurance contract it is furthermore assumed that they are purely financial assets traded on a perfectly competitive, complete, frictionless, transparent and arbitrage-free market, driven by rational investors all sharing the same information.

C.6.12 Since the liability from a life insurance contract usually contains non-hedgeable risk factors (indicating market incompleteness) the valuation is more problematic. Comparing non-hedgeability in life insurance contracts with the assumptions behind the Black-Scholes option pricing formula on can conclude that almost all the assumptions are violated (not a complete list):

- There is no longer a unique self-financing trading strategy;
- Many possible hedging-strategies exists, where no one is perfect;
- It will not be possible to rebalance the replicating portfolio on a continuous basis (discrete rebalancing).
- The dynamics of the risky asset might not be correctly specified.
- The geometric Brownian motion assumption is not a realistic assumption at least for interest rates dynamics;
- There are transaction costs on trading;
- Incomplete financial markets (long time maturity of the life insurance contracts cause absence of real risk-free asset and suitable options, complicated exercise behaviour of options in financial products, absence of mortality linked instruments etc.)
- The underlying reference portfolio might consist of illiquid assets and therefore it might not be a tradable asset;
- Due to market incompleteness hedging can be very costly;
- The volume of the trading does affect the price;
- In real life a portfolio cannot be infinitely divisible;
- For long-dated options constant volatility is an unrealistic assumption and should be replaced with stochastic volatility;
- How should policyholder behaviour be "traded"?
- How should management discretion and actions be "traded"?

C.6.13 Applying the Black-Scholes option pricing formula could thus give an illusion of a risk-free position. If the actual investment strategy (self-hedge), the portfolio characteristics and other assumptions are not in accordance with the Black-Scholes framework, risk is still present.

C.6.14 Very few with-profits guarantees can be appropriately modelled with the use of the Black-Scholes options pricing formula, or with the Black formulas for interest rate options. However, the other alternative modelling approaches requires a rather sophisticated modelling, which may not be practicable for all firms.

C.6.15 Applying the Black-Scholes based option pricing formulas could nevertheless give valuable first insights to the costs and be seen as an interim approach until a more appropriate approach have been created.

Especially small firms might never be able to achieve non-formula estimates and therefore formulas like the Black-Scholes option pricing formula could be used. However, the drawbacks and the limitations of applying closed-form estimates within a Black-Scholes framework should be recognised.

C.7 Investment guarantees

C.7.1 Assume an equity-indexed (unit-) linked life insurance contract with a single premium and a payment $G(T)$ at maturity T that is guaranteed at issue of the contract. At maturity the policyholder will receive an amount corresponding to the index but not less than the guaranteed amount. Assume further complete markets, no expenses and that the contract cannot be surrender before maturity and that no benefit is paid upon death (the contract will always be held to maturity). The random payout at maturity $\tilde{V}(T)$ is then given by the formula

$$\tilde{V}(T) = \max\{\tilde{S}(T), G(T)\} \ ,$$

where $\tilde{S}(T)$ the random value of the equity-index at time T. From the put-call parity for European options (assuming thus implicitly a Black-Scholes framework) the random payout at maturity can be written as

$$\tilde{V}(T) = \begin{cases} G(T) + \max\{\tilde{S}(T) - G(T);0\} & \text{(Call option)} \\ \tilde{S}(T) + \max\{G(T) - \tilde{S}(T);0\} & \text{(Put option)} \end{cases} .$$

C.7.2 By multiplying with the T-year risk-free discount factor and taking the expectation of the random variable $\tilde{V}(T)$ with respect to the risk-neutral measure Q the following equations for the liability are obtained

$$L = E_Q[e^{-r_f(T)\cdot T} \cdot \tilde{V}(T)] = \begin{cases} e^{-r_f(T)\cdot T} \cdot G(T) + E_Q[e^{-r_f(T)\cdot T} \cdot \max\{\tilde{S}(T) - G(T);0\}] & \text{(Call option)} \\ E_Q[e^{-r_f(T)\cdot T} \cdot \tilde{S}(T)] + E_Q[e^{-r_f(T)\cdot T} \cdot \max\{G(T) - \tilde{S}(T);0\}] & \text{(Put option)} \end{cases} .$$

C.7.3 Let $G(0) = e^{-r_f(T)\cdot T} \cdot G(T)$ denote the present value of the guarantee and $S(0) = E_Q[e^{-r_f(T)\cdot T} \cdot \tilde{S}(T)]$ denote the current spot market value of the equity-index (comes from the nature of a risk-neutral valuation). The liability L can thus be written as

$$L = \begin{cases} G(0) + E_Q[e^{-r_f(T)\cdot T} \cdot \max\{\tilde{S}(T) - G(T);0\}] & \text{(Call option)} \\ S(0) + E_Q[e^{-r_f(T)\cdot T} \cdot \max\{G(T) - \tilde{S}(T);0\}] & \text{(Put option)} \end{cases}$$

or

$$L = \begin{cases} \text{Guarantee} + \text{Intrinsic value of extra benefits} + \text{Option time value} & \text{(Call option)} \\ \text{Underlying asset} + \text{Intrinsic value of guarantee} + \text{Option time value} & \text{(Put option)} \end{cases}.$$

C.7.4 In the call option approach the intrinsic value of extra benefits corresponds to the amount the call option is in-the-money if it would be exercise immediately. The option time value captures the potential to receive further extra benefits in the future due to the random fluctuations of the underlying asset, causing additional costs for the firm.

C.7.5 In the put option approach the intrinsic value of guarantee corresponds to the amount the guarantee is in-the-money if it would be exercised immediately. The time value captures the potential for the cost to change in value (guarantee to bite further) in the future, as the guarantee move (related to the variability of the underlying asset) into or out-of-the money, causing additional costs for the firm.

C.7.6 Thus, under certain economic scenarios additional shareholder or other contributions are required to meet the policyholder's benefit payments and the average additional cost of these events forms the time value of the guarantee. Obviously, the time value of guarantees has to be determined using stochastic modelling techniques.

C.7.7 The fundamental difference between the call and put option approach is that the call option takes into account the variability of investment profits that will be above the intrinsic value and the put option approach takes into account the variability of investment losses that will make the guarantee bite more than its intrinsic value. This will be an important issue when incorporating management discretion into the valuation model, as it is easier to cut excess profits than to cut losses. Therefore the placeholder approach to determine the time value is the put option approach. However, in a Black-Scholes framework and quite often in practise the time value in a call or put option approach are equal (symmetric process).

C.7.8 The intrinsic value is commonly estimated by using representative deterministic assumptions of the possible future outcome. This is commonly done by a reference to the term structure of risk-free interest rates. However, in more complicated situations where for instance the valuation is affected by management actions with perhaps an intention to distribute excess capital to policyholders care has to be taken in order to appropriately take these kinds of aspects into account.

C.7.9 Assume now an equity-indexed (unit-) linked life insurance contract with a single premium and an annual constant guaranteed rate r_g (continuous) for all times $i \leq T$ that is guaranteed at the issue of the contract. The policyholder will annually receive a random return $\tilde{r}_I(i)$ (continuous) corresponding to the return on the index but not less than the guaranteed return r_g. Again, assume complete markets, no expenses and that the contract cannot be surrender before maturity and that no benefit is paid upon death (the contract will always be held to maturity).

C.7.10 The annual random accumulation factor to the policyholder at time i is thus (assuming again implicitly a Black-Scholes framework) given by

$$e^{\tilde{r}_p(i)} = 1 + \max\{e^{\tilde{r}_f(i)} - 1; e^{r_g} - 1\} = \begin{cases} e^{r_g} + \max\{e^{\tilde{r}_f(i)} - e^{r_g}; 0\} & \text{(Call option)} \\ e^{\tilde{r}_f(i)} + \max\{e^{r_g} - e^{\tilde{r}_f(i)}; 0\} & \text{(Put option)} \end{cases} .$$

C.7.11 Note that for a specific year the situation is similar to the guaranteed minimum maturity benefit described earlier. Let $V(0)$ denote the (deterministic) retrospective reserve or the policy fund at time zero (time of valuation).

C.7.12 The random policy reserve $\tilde{V}(t)$ at time t develops in a call option approach as

$$\tilde{V}(t) = \tilde{V}(t-1) \cdot \left(e^{r_g} + \max\{e^{\tilde{r}_f(t)} - e^{r_g}; 0\}\right) = V(0) \cdot \prod_{i=1}^{t} \left(e^{r_g} + \max\{e^{\tilde{r}_f(i)} - e^{r_g}; 0\}\right)$$

and in a put option approach as

$$\tilde{V}(t) = \tilde{V}(t-1) \cdot \left(e^{\tilde{r}_f(t)} + \max\{e^{r_g} - e^{\tilde{r}_f(t)}; 0\}\right) = V(0) \cdot \prod_{i=1}^{t} \left(e^{\tilde{r}_f(i)} + \max\{e^{r_g} - e^{\tilde{r}_f(i)}; 0\}\right)$$

.

C.7.13 By multiplying with the T-year risk-free discount factor and taking the expectation of the random variable $\tilde{V}(T)$ with respect to the risk-neutral measure Q the following equation for the liability is obtained in the call option approach

$$L = E_Q[e^{-r_f(T) \cdot T} \cdot \tilde{V}(T)] = V(0) \cdot E_Q\left[e^{-r_f(T) \cdot T} \cdot \prod_{i=1}^{T} \left(e^{r_g} + \max\{e^{\tilde{r}_f(i)} - e^{r_g}; 0\}\right)\right]$$

and in a put option approach

$$L = E_Q[e^{-r_f(T) \cdot T} \cdot \tilde{V}(T)] = V(0) \cdot E_Q\left[e^{-r_f(T) \cdot T} \cdot \prod_{i=1}^{T} \left(e^{\tilde{r}_f(i)} + \max\{e^{r_g} - e^{\tilde{r}_f(i)}; 0\}\right)\right].$$

C.7.14 Under classical Black-Scholes assumptions the random return from consecutive years $t = 1,2,\ldots$ are independent and identically distributed under the unique Q-measure. Thus we can move the expectation into the product stream and calculate the expectation of each term separately. After rewriting

$$e^{-r_f(T) \cdot T} = e^{-f(0,1)} \cdot e^{-f(1,2)} \cdots e^{-f(T-1,T)} = \prod_{i=1}^{T} e^{-f(i-1,i)} \quad ,$$

where $f(i-1,i)$ denotes the annual continuous forward rate from time $i-1$ to i derived from the term structure of risk-free interest rates. The liability can be written in a call option approach as

$$L = E_Q[e^{-r_f(T)\cdot T} \cdot \tilde{V}(T)] = V(0) \cdot \prod_{i=1}^{T} E_Q\left[e^{-f(i-1,i)} \cdot \left(e^{r_g} + \max\{e^{\tilde{r}_I(i)} - e^{r_g};0\}\right)\right]$$

and in a put option approach as

$$L = E_Q[e^{-r_f(T)\cdot T} \cdot \tilde{V}(T)] = V(0) \cdot \prod_{i=1}^{T} E_Q\left[e^{-f(i-1,i)} \cdot \left(e^{\tilde{r}_I(i)} + \max\{e^{r_g} - e^{\tilde{r}_I(i)};0\}\right)\right] .$$

C.7.15 Taking the expectations and assuming a Black-Scholes framework yields the following expression for the call option approach

$$L = V(0) \cdot \prod_{i=1}^{T} \left(e^{r_g - f(i-1,i)} + \Phi(d_1(i)) - e^{r_g - f(i-1,i)} \cdot \Phi(d_2(i))\right)$$

and for the put option approach

$$L = V(0) \cdot \prod_{i=1}^{T} \left(1 + e^{r_g - f(i-1,i)} \cdot \Phi(-d_2(i)) - \Phi(-d_1(i))\right) ,$$

where $\Phi(x)$ denotes the cumulative standard normal distribution function and

$$d_1(i) = \frac{\ln(1/e^{r_g}) + f(i-1,i) + \sigma^2/2}{\sigma} \qquad \text{and} \qquad d_2(i) = d_1(i) - \sigma .$$

C.7.16 Hence the liability shows the expected amount that should be held in addition to the underlying assets to be able to deliver the benefits (due to the guarantee).

C.7.17 Introducing management actions and discretion into the valuation complicates its considerable. The Black-Scholes framework introduced above assumes path-independent accumulation factors to the policyholder. In practise past investment returns, decisions and especially the solvency position of the firm will usually have a significant impact on the accumulation factor and create complex path-dependent processes not suitable for closed-form modelling. Most likely, simplified approaches have to be adopted by small firms.

C.7.18 The situation could be simplified, at least initially for an interim period, by assuming that the a firm's principle and practice for distributing extra benefits can be described in a one single constant distribution ratio factor β of the future random investment returns. Then closed-form solutions can be obtained. Define the annual random accumulation factor to the policyholder at time i as

$$e^{\tilde{r}_p(i)} = 1 + \max\{\beta \cdot (e^{\tilde{r}_I(i)} - 1)\,;\, e^{r_g} - 1\} = \begin{cases} e^{r_g} + \beta \cdot \max\{e^{\tilde{r}_I(i)} - \dfrac{e^{r_g} - (1-\beta)}{\beta};0\} & \text{(Call option)} \\ 1 + \beta \cdot (e^{\tilde{r}_I(i)} - 1) + \beta \cdot \max\{\dfrac{e^{r_g} - (1-\beta)}{\beta} - e^{\tilde{r}_I(i)};0\} & \text{(Put option)} \end{cases}.$$

C.7.19 Performing the same reasoning as above we obtain for the call option approach

$$L = V(0) \cdot \prod_{i=1}^{T} \left(e^{r_g - f(i-1,i)} + \beta \cdot \left[\Phi(d_1(i)) - e^{-f(i-1,i)} \cdot \frac{e^{r_g} - (1-\beta)}{\beta} \cdot \Phi(d_2(i)) \right] \right)$$

and for the put option approach

$$L = V(0) \cdot \prod_{i=1}^{T} \left(e^{-f(i-1,i)} + \beta \cdot (1 - e^{-f(i-1,i)}) + \beta \cdot \left[e^{-f(i-1,i)} \cdot \frac{e^{r_g} - (1-\beta)}{\beta} \cdot \Phi(-d_2(i)) - \Phi(-d_1(i)) \right] \right) ,$$

where $\Phi(x)$ denotes the cumulative standard normal distribution function and

$$d_1(i) = \frac{\ln\left(\dfrac{\beta}{e^{r_g} - (1-\beta)} \right) + f(i-1,i) + \sigma^2 / 2}{\sigma} \quad \text{and} \quad d_2(i) = d_1(i) - \sigma \ .$$

C.7.20 It should be noted that due to the symmetry the call and put option approach will again give the same answers.

C.7.21 Thus by introducing a constant distribution ratio the guaranteed benefits would remain on the same level in the call option approach but the amount of extra benefits expected to be distributed would be reduced. In the put option approach the value of the underlying assets would be reduced and there would also be a reduction effect on the option value.

C.7.22 It should be noted that changing the underlying asset allocation (more or less volatility) or the investment strategy relative to the Black-Scholes framework the value of the liability for the firm would probably change.

C.8 Other options and guarantees

C.8.1 Many other types of options and guarantees are commonly included in life insurance contracts. Guarantees including those that have been mentioned earlier are for instance (the illustrations are not necessarily mutually exclusive):

- guaranteed minimum surrender values in with-profits contracts;
- guaranteed minimum death benefits in with-profits contracts;
- guaranteed minimum maturity benefit in with-profits contracts;

- guaranteed minimum annuity rates in with-profits contracts;
- guaranteed benefits that depends on the payment of all future premiums;
- guaranteed benefits that depends on some pre-specified time;
- guaranteed minimum investment return in with-profits contracts;
- guaranteed multi-step investment return in with-profits contracts;
- guaranteed minimum investment return on declared extra benefits (reinvestment guarantee) in with-profits contracts;
- guaranteed mortality charge basis in with-profits contracts;
- guaranteed expense charge basis in with-profits contracts;
- guaranteed structure for distributing extra benefits (reversionary, terminal etc.).

C.8.2 Policyholder options include for instance

- right to surrender;
- right to withdraw (partial surrender);
- right to pay flexible premiums with respect of time or amount;
- right to increase life cover subsequently without underwriting;
- right to convert from one policy to another;
- right to convert between a guaranteed annuity and a lump sum benefit;
- right to switch funds from an traditional with-profits contract to an unit-linked contract (and vice versa);
- right to lock in profits.

C.8.3 Firm options include for instance

- reduction in extra benefits;
- change of asset allocation;
- change of investment strategy;
- allocation of extra benefits between reversionary and terminal benefits;
- right to refuse premiums from existing contracts;

- right to change valuation basis (mortality, expenses, guaranteed interest rate, other charges, market value reductions etc.).

C.8.4 Obviously, it is rather impossible to give detailed valuation approaches that would be suitable for all possible options and guarantees. However, some of them can be valued with similar techniques as those for the surrender option and some of them can be valued with similar techniques as those for the investment guarantee.

C.8.5 One possible simplification for expense and mortality guarantees in with-profits insurance contracts would be to include them into the valuation of the investment guarantee for instance as an increment of profits distributed as extra benefits relative to the investment return (increment of the distribution ratio for extra benefits).

C.8.6 For other options and guarantees where the surrender option valuation approach cannot be sensible applied a last resort would be a subjective ad hoc valuation. The first step would then be to analyse the characteristics of the option or the guarantee and how it would probable effect the cash-flows. The second step would be to analyse the amount the option or guarantee is expected to be currently in-the-money. The third step would be to determine how much the cost of the option or the guarantee is expected to vary as time passes and the last step would be to estimate the probability that the cost of the option or the guarantee would become more costly in the future. The total cost of the option or the guarantee would then be crudely approximated as a subjective expected intrinsic value increased with a subjective expectation of future variation of cost, which could be estimated as the probability for the option or guarantee to become more valuable in the future times the expected cost for that event.

C.8.7 As an interim approach one would clearly have to focus on the most material guarantees and options and is may be useful as for many other areas to adopt an iterative development process where approximations are gradually improved.

C.8.8 Quite often the valuation of options and guarantees are considered to be the most challenging part in the valuation of the technical provision. Indeed they create difficulties, but of equal difficulty is also an appropriate valuation of extra benefits.

C.9 Distribution of extra benefits

C.9.1 The large influence of management discretion in the valuation of technical provision for with-profits business raise a number of important issues that needs to be given appropriate attention in order to have sufficient confidence in the valuation and achieve efficient supervision of the valuation. An accurate assessment and a detailed enough documentation of the mechanism for distribution extra benefits forms the cornerstones. Since the distribution of extra benefits plays a central role for firms with a significant amount of with-profits business this mechanism will probable encompass a significant amount of the spectrum of principles and practices a firm has adopted to run the business. Furthermore, the mechanism is also strongly related to the financial position of the firm, which is often set as a primary restriction for distribution of extra benefits.

C.9.2 Some key issues (not necessary mutually exclusive) in the mechanism for distribution extra benefits are the following (should in most cases be set for a homogenous group of policyholders even if so not explicitly stated):

- What constitutes a homogenous group of policyholders and what are the key drivers for the grouping?
- How is a profit divided between owners of the firm and the policyholders and furthermore between different policyholders?
- How is a deficit divided between owners of the firm and the policyholders and furthermore between different policyholders?
- How will the mechanism for extra benefits be affected by a large profit or loss?
- How will policyholders be affected by profits and losses from other activities?
- What is the target return level set by the firm's owners on their invested capital?
- What are the key drivers affecting the level of extra benefits?
- What is an expected level (inclusive any distribution of excess capital, unrealised gains etc.) of extra benefits?
- How are the extra benefits made available for policyholders and what are the key drivers affecting for example the split between reversionary and terminal extra benefits, conditionality, changes in smoothing practise, level of discretionary by the firm etc.
- How will the experience from current and previous years affect the level of extra benefits?
- When is a firm's solvency position so weak that declaring extra benefits is considered by the firm to be jeopardizing a firm-owner's or/and policyholders' interest?
- What other restrictions are in place for determining the level of extra benefits?
- What is a firm's investment strategy?
- What is the asset mix driving the investment return?

- What is the smoothing mechanism if used and what is the interplay with a large profit or loss?

- What kind of restrictions are in place in smoothing extra benefits?

- Under what circumstances would one expect significant changes in the crediting mechanism for extra benefits?

- To what extent is the crediting mechanism for extra benefits sensitive to policyholders' actions?

C.9.3 As for any other assumption a comprehensive analysis of past experience, practise and crediting mechanism is prerequisite for an appropriate valuation of technical provisions. However, the crediting mechanism is not expected to be static and even if it should be sufficiently stable over time it may be subject to changes.

C.9.4 For many firms an implementation of a broad crediting mechanism in the valuation of technical provisions will probable, at least initially, be out of reach. When using more approximate methods such as a distribution ratio factor of the investment return care has to be taken not to underestimate the factor, since this would underestimate the whole liability. Nevertheless, excess prudence should not be included. The balance between a sensible factor and excess prudence is not an easy task, but any choice of a factor should be properly justified.

C.10 Expenses and other charges

Expenses

C.10.1 In life insurance contracts there is commonly an explicit loading for expenses that is charged from the premiums or policyholder's fund. Therefore the best estimate benefit cash-flows do not recognise the future expected expenses and a separate liability for expenses needs to be set up.

C.10.2 The general principle states that the present value of contract loadings and the present value of expected expenses should be recognised explicitly in the cash-flow projections. Any shortfall would need to be recognised as an additional liability in order to ensure that expenses required to manage the business are reflected in total.

C.10.3 Under a stochastic simulation approach asset and liability cash flows are projected under a variety of financial market scenarios that should be consistent with the investment strategy the firm has chosen. Expenses to be incurred should thus explicitly be included in the simulation and the future expense inflation should be consistent with what is assumed in the interest rate scenario and other relevant factors influencing the expenses. In some cases, both the future expenses and the expense loadings may be sensitive to changes in inflation. However, one should not assume them to equal each other unless there is proper evidence of such immunisation.

C.10.4 As for any other assumption, the estimation of the best estimate assumption for expenses begins with an analysis of existing firm specific experience. The aim of the analysis is to obtain an understanding of current and historical expenses that in addition to absolute amounts also includes an analysis of for instance where expenses occur (functions, processes, business segments, products etc.), factors that influence the expenses and how the expenses are related to sizes and natures of insurance portfolios.

C.10.5 A good starting point is the split of expenses in different functions in the annual accounts:

- Acquisition expenses;
- Administration expenses;
- Maintenance expenses;
- Claims expenses; and
- Investment expenses.

C.10.6 Since not all of the expenses are relevant for the valuation of the expense liability considerations has to be given to which expenses should be excluded. These typically include marketing and acquisition expenses, product development expenses, parts of administration expenses etc. It is of special importance to identify the expenses that are sensitive to inflation (e.g. policy maintenance expenses).

C.10.7 An expense analysis is commonly based on a single financial year of the firm. In order to appropriately take into account trends and to be able to ensure that recent changes and trends in expense levels are reflected appropriately several financial years should be included in the analysis. Of importance is also that each year is consistently analysed.

C.10.8 The approach to value the expense liability relies on the existence of model that projects the expenses into the future consistently with other cash-flows. This requires a rather sophisticated modelling that might not be possible or in place for all firms. Therefore the possibility of a more pragmatic approach is needed.

C.10.9 Most firms engage themselves either voluntary or upon a request from the supervisory authority in an annual so-called calculation base analysis, where the risk, expense and discount/valuation interest rate assumptions are compared with outcomes experienced. The expense analysis together with the expense loadings generated from the future cash-flows could form a basis for the valuation of the expense liability.

C.10.10 The first step would be to exclude from the data those expenses and expense loadings (e.g. upfront acquisition charges) that should not be included in the liability. The second step would be to perform the same procedure consistently to a number of consecutive years (say *n* years). The third step would be to calculate the present value of future expected expense loadings. The expense liability (*EL*) could thereafter be approximated by

$$EL \approx \text{Present value of future expected expense loadings} \cdot \frac{1}{n} \cdot \sum_{i=0}^{n-1} \frac{e_{incurred}(t-i)}{e_{loading}(t-i)} \ ,$$

C.10.11 where $e_{incurred}(k)$ denotes the incurred expenses reduced with irrelevant expenses in year k and $e_{loading}(k)$ denotes the expense loadings reduced with irrelevant expenses in year k and t denotes the latest financial year. If the future expense loadings are sufficient no additional reserves needs to be set up (and the opposite). Note that historical inflation and the difference in the sensitivity of inflation between incurred expenses and expense loadings are to some extent implicitly included in the n-year average of the expense ratio.

Other charges

C.10.12 Classical financial theory on derivative pricing assumes that the cost of an option is charged in advance that is at the time off contract issue. However, for life insurance contracts with embedded options (especially for with-profits contracts) it is rather common that for the cost of the embedded options only a minor charge is made up front and that the remainder is due to over an extended period of time (not necessary to total time until maturity and not necessarily fixed or known exactly in advance).

C.10.13 The deferred charges could give the policyholder a long position in an option to terminate payments or surrender the contract without having paid the full value of the embedded options to the firm which has a short position. A rational policyholder would surrender the contracts when it becomes uneconomical, that is a rational policyholder would take full benefits from the embedded options and lapse the policy.

C.10.14 Commonly the loss is shared among other policyholders belonging to the same homogenous group or sometimes even wider among different groups of policyholders. It should be noted that a transfer of costs between different groups of policyholders can be restricted (in law or in some other way).

C.10.15 Another interesting situation arises when there is a shift from single-premium to periodic premiums contracts. The probability of paying an extra premium is in general dependent on economic variables that also affect the surrender option and since these also affect the risks associated with deferred charges related to the cost of embedded options, a complicated path dependent environment have been created that cannot be solved in a closed form.

C.10.16 The ability to surrender creates an incomplete market situation in which the liability cannot be perfectly hedged. In some sense deferring charges from embedded options and guarantees completes the market. However, no fair deferred charging structure could be such that it would complete the market in full and therefore the ability to surrender always causes a non-hedgeable position. If unfair the deferred charges would be so large that they would with a very high probability always be enough to cover the cost from embedded options, but even so the exposure would still be non-hedgeable.

C.10.17 The cost of embedded options can be charged in a various ways, such as

- fixed upfront charge reduced from paid premiums,
- market consistent upfront charge reduced from paid premiums,
- fixed periodic (annual) charge proportional to policyholder's fund value,
- market consistent periodic (annual) charge,
- fixed periodic (annual) charge proportional to the total distributional amount of extra benefits (reduction in extra benefits in with-profit contracts);
- an arbitrary charge level (a form of retrospective price adjustment by a refunding of incurred costs, unfixed variable charge rates etc.) at the discretion by the firm taken into account as a reduction of extra benefits in with-profit contracts; or
- any combination of the above.

C.10.18 Charges from embedded options should be taken into account in the best estimate valuation of technical provisions and they should be kept separately from expense loadings. A surrender charge could possible be seen as a charge to in average offset the uncollected charges, but could also be seen as way to force the policyholder to continue the contract and hence it would not directly be related to the cost of embedded options.

C.10.19 It should be noted that some charging structures for embedded options are transparently disclosed in the valuation basis for a product, whereas some charging structures are disclosed in a firm's principles and practices to run the business (public or not). In such cases the valuation should be consistent with set principles. For firms applying the second last approach the principles and practices are not transparently disclosed and the structure resembles undisclosed management actions. It should be noted that without any real documentation, it will be very difficult to have any confidence on the valuation of technical provisions.

C.10.20 If the charges can be explicitly valued and taken into account this should so be done in the valuation of technical provisions. For less advanced firms a rather pragmatic approach might nevertheless have to be adopted. Charges from embedded options could in such cases be defined as a fixed percentage of policyholder's fund value or as a fixed reduction in extra benefits. In such cases a proper analysis has to be carried in order to ensure a meaningful estimation of the parameters. In practice firms may face complex situations where both the policyholder and the firm can have different types of options which may end up being in the money, and the net effect of these options needs to be somehow assessed. The first step is of course a clear understanding of all the options that are embedded in each product line.

C.11 Illustrative example A

C.11.1 The following example is based on successive applications of the Black-Scholes model (as explained above) to Thiele's equation that governs the change of technical provision. It is only for illustrative purposes and a number of simplifications have been made in order to obtain more accessible formulas. However, the approach can rather easily be enlarged to more complicated situations that may arise in practise, and by reducing the discretisation time step, increased accuracy can be obtained. If certain symbols are not explicitly defined here, they can be reviewed from the earlier sections.

C.11.2 Assume as a starting point a discretised deterministic Thiele's differential equation[4], which in general terms can be written (after a suitable grouping) as

$$V(t) = e^{r_g} \cdot V(t-1) + e^{r_g} \cdot D_1(t-1) + V(t-1) \cdot D_2(t-1) + D_3(t-1) \ ,$$

where $D_1(t-1)$, $D_2(t-1)$ and $D_3(t-1)$ are deterministic and where each of them is further a collection of deterministic terms. Introducing a stochastic interest rate guarantee given by

$$e^{\tilde{r}_p(t)} = 1 + \max\{\beta \cdot (e^{\tilde{r}_f(t)} - 1); e^{r_g} - 1\}$$

(see the section on investment guarantees above), transforms Thiele's equation above to

$$\tilde{V}(t) = e^{\tilde{r}_p(t)} \cdot \tilde{V}(t-1) + e^{\tilde{r}_p(t)} \cdot D_1(t-1) + \tilde{V}(t-1) \cdot D_2(t-1) + D_3(t-1) \ .$$

C.11.3 Multiplying by the discount factor and taking the expectation yields an approximation for the present value of the expected fund value FV at time t

$$FV(t) = E[e^{-r_f(t)\cdot t} \cdot \tilde{V}(t)] = E[e^{-r_f(t)\cdot t} \cdot e^{\tilde{r}_p(t)} \cdot \tilde{V}(t-1)] + D_1(t-1) \cdot E[e^{-r_f(t)\cdot t} \cdot e^{\tilde{r}_p(t)}]$$

$$+ D_2(t-1) \cdot E[e^{-r_f(t)\cdot t} \cdot \tilde{V}(t-1)] + E[e^{-r_f(t)\cdot t} \cdot D_3(t-1)] \ .$$

C.11.4 Now, by assuming a Black-Scholes framework

[4] The time period has been set equal to one year.

$$E[e^{-r_f(t)\cdot t}\cdot e^{\tilde{r}_P(t)}\cdot\tilde{V}(t-1)] = E[(e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)})\cdot(e^{-r_f(t-1)\cdot(t-1)}\cdot\tilde{V}(t-1))]$$
$$= E[e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)}]\cdot E[e^{-r_f(t-1)\cdot(t-1)}\cdot\tilde{V}(t-1)]$$

$$D_1(t-1)\cdot E[e^{-r_f(t)\cdot t}\cdot e^{\tilde{r}_P(t)}] = D_1(t-1)\cdot e^{-r_f(t-1)\cdot(t-1)}E[e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)}]$$

$$D_2(t-1)\cdot E[e^{-r_f(t)\cdot t}\cdot\tilde{V}(t-1)] = D_2(t-1)\cdot e^{-f(t-1,t)}\cdot E[e^{-r_f(t-1)\cdot(t-1)}\cdot\tilde{V}(t-1)]$$

$$E[e^{-r_f(t)\cdot t}\cdot D_3(t-1)] = e^{-r_f(t)\cdot t}\cdot D_3(t-1)$$

and hence

$$FV(t) = E[e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)}]\cdot E[e^{-r_f(t-1)\cdot(t-1)}\cdot\tilde{V}(t-1)] + D_1(t-1)\cdot e^{-r_f(t-1)\cdot(t-1)}\cdot E[e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)}]$$
$$+ D_2(t-1)\cdot e^{-f(t-1,t)}\cdot E[e^{-r_f(t-1)\cdot(t-1)}\cdot\tilde{V}(t-1)] + e^{-r_f(t)\cdot t}\cdot D_3(t-1)$$
$$= E[e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)}]\cdot FV(t-1) + D_1(t-1)\cdot e^{-r_f(t-1)\cdot(t-1)}\cdot E[e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)}]$$
$$+ D_2(t-1)\cdot e^{-f(t-1,t)}\cdot FV(t-1) + e^{-r_f(t)\cdot t}\cdot D_3(t-1),$$

where

$$E[e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)}] = e^{r_g-f(t-1,t)} + \beta\cdot\left[\Phi(d_1(i)) - e^{-f(t-1,t)}\cdot\frac{e^{r_g}-(1-\beta)}{\beta}\cdot\Phi(d_2(i))\right]$$

and $\Phi(x)$ denotes the cumulative standard normal distribution function and

$$d_1(i) = \frac{\ln\left(\dfrac{\beta}{e^{r_g}-(1-\beta)}\right) + f(t-1,t) + \sigma^2/2}{\sigma} \quad \text{and} \quad d_2(i) = d_1(i) - \sigma\ .$$

C.11.5 Thus, after the expectations $E[e^{-f(t-1,t)}\cdot e^{\tilde{r}_P(t)}]$ and $E[e^{-r_f(t)\cdot t}\cdot\tilde{V}(t)]$ have been derived for all t the fund values can be estimated from which the probability weighted benefits liability can be estimated as

$$L = \sum_{t=1}^{T} {}_{t-1}p_x\cdot{}_1q_{x+t-1}\cdot{}_tp_0^s\cdot CFD(t) + \sum_{t=1}^{T} {}_tp_x\cdot{}_{t-1}p_0^s\cdot{}_tq_{t-1}^s\cdot CFS(t)$$
$$+ \sum_{t=1}^{T} {}_{t-1}p_x\cdot{}_1q_{x+t-1}\cdot{}_{t-1}p_0^s\cdot{}_tq_{t-1}^s\cdot CFSD(t) + {}_Tp_x\cdot{}_Tp_0^s\cdot CFM(T)$$

where $CFD(t)$ is the cash-flow at time t (at the end of the year) from a death event, $CFS(t)$ is the cash-flow at time t from a surrender event, $CFSD(t)$ is the cash-flow at time t from a death and a surrender event and $CFM(T)$ is the cash-flow from a no death and a no surrender event up to time T.

C.11.6 Furthermore ${}_{t-1}p_x$ and ${}_{1}q_{x+t-1}$ are classical death and survival probabilities and ${}_{t-1}p_0^s$ and ${}_{t}q_{t-1}^s$ are corresponding no-surrender and surrender probabilities (mortality and surrenderability is thus assumed to be modelled in the same way, mutually independent and independent of the financial market).

C.11.7 Consider now a with-profits life insurance contract where at time zero (the beginning of year one) the policyholder pays a premium of 1000 to the firm. The policy matures after 5 years after which the firm makes a single payment to the policyholder. The contract can also be terminated earlier depending on policyholder's preferences before maturity. If the insured dies before maturity of the contract a death benefit of 105 % (denoted by $1+z$) of the fund value is paid. Through out the contract the firm guarantees the valuation basis for guaranteed interest rate, mortality, expenses and also other expenses. Thus, for any given time between now and maturity a guaranteed minimum benefit can be calculated upon surrender, death or maturity. Periodic premiums are contractually agreed to be 100 per year.

C.11.8 The valuation basis consist of the following assumptions (c.f. the more detailed specific sections above):

- Mortality, surrender and the financial markets are independent from each other.
- The premiums are assumed to be paid at the beginning of each year.
- The extra benefits relative to the investment return per annum is estimated to be 89 %. From the profits due to prudent mortality assumptions a further 1 % is added and no profit is distributed as extra benefits from the expense assumptions. Thus the total distribution ratio for extra benefits β equals 90 %.
- The historical average expense ratio equals 105 % thus indicating a 5 % deficiency relative to the expense loadings.
- Charges for costs of the guarantees (denoted by γ_2) are 0.5 % per annum and proportional to the fund value.
- Expense charges proportional to the fund value (denoted by γ_1) are 1.5 % per annum and expense charges proportional to the premiums (denoted by κ) are 2 % .
- There is a 1 % expense charge from the fund value upon surrender.
- The guaranteed interest rate r_g equals 1 % per annum.
- The force of mortality in the valuation basis follows a Gompertz-model:

$$\mu(x,t) = \frac{1}{12} \cdot e^{\frac{x+t-83}{12}} .$$

- The surrender intensity is given by $\gamma_t = 0.3 - \frac{0.1}{t+10}$.
- The best estimate force of mortality is $\mu_{BE}(x,t) = 0.8 \cdot \mu(x,t)$.
- The volatility σ of the underlying reference portfolio is a constant 7.5 % per annum.
- The age x of the policyholder at the inception of the contract is 70.
- The number of years between the inception of the contract and time until maturity is exactly 5 years.
- All future benefits such as death, surrender or terminal value payments take place at integer payment dates 1,2,3,4 or 5.
- It is assumed that the insured and the policyholder is the same person and if both death and surrender occur in the same time period the contract was surrendered.
- No other options and guarantees exist.
- The observed interest rates are:

Year (*t*)	**Zero interest rate for an *t*-year investment**	**One year forward rate for the *t*:th year**
1	3,064 %	
2	3,461 %	3,861 %
3	3,702 %	4,185 %
4	3,905 %	4,516 %
5	4,096 %	4,861 %

C.11.9 Assume further that the stochastic policy fund value $\widetilde{V}(t)$ at time t evolves with an annual period according to[5]

$$\widetilde{V}(t) = e^{\tilde{r}_p(t)} \cdot (\widetilde{V}(t-1) + B(t-1)) - [(\gamma_1 + \gamma_2) + \mu(x,t-1) \cdot z] \cdot (\widetilde{V}(t-1) + B(t-1)) ,$$

with $B(0) = 0$. Multiplying by the discount factor and taking the expectation yields an approximation for the present value of the expected fund value FV at time t (= 1,2,3,4 and 5)

[5] $\widetilde{V}(t) = a \cdot e^{\tilde{r}_p(t)} \cdot (\widetilde{V}(t-1) + B(t-1)) + a \cdot \mu(x,t-1) \cdot (\widetilde{V}(t-1) + B(t-1)) - a \cdot (\gamma_1 + \gamma_2) \cdot (\widetilde{V}(t-1) + B(t-1))$
$- a \cdot \mu(x,t-1) \cdot (1+z) \cdot (\widetilde{V}(t-1) + B(t-1))$

$B(0) = 0$ and the period $a = 1$ (monthly $a = 1/12$).

$$FV(t) = E[e^{-r_f(t)\cdot t} \cdot \widetilde{V}(t)] = E[e^{-f(t-1,t)} \cdot e^{\widetilde{r}_p(t)}] \cdot E[e^{-r_f(t-1)\cdot(t-1)} \cdot \widetilde{V}(t-1)]$$

$$+ e^{-r_f(t-1)\cdot(t-1)} \cdot (1-\kappa) \cdot B(t-1) \cdot E[e^{-f(t-1,t)} \cdot e^{\widetilde{r}_p(t)}]$$

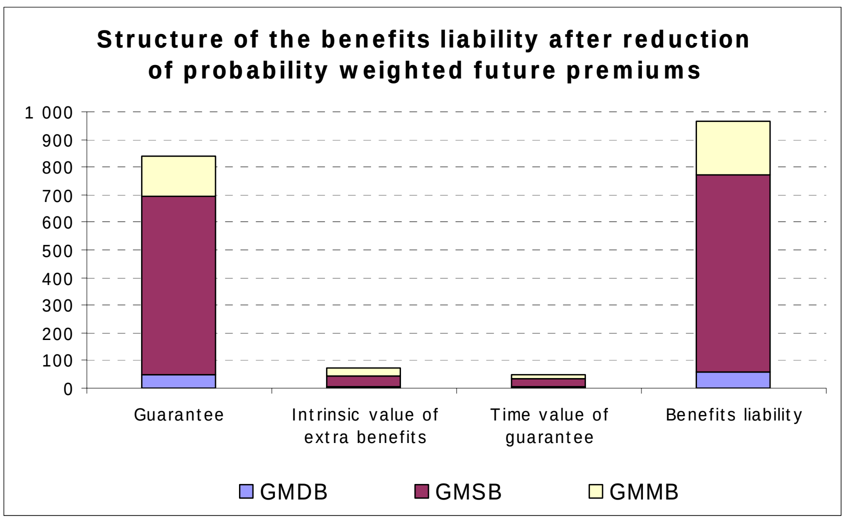
$$- [(\gamma_1 + \gamma_2) + \mu(x, t-1) \cdot z] \cdot e^{-f(t-1,t)} \cdot E[e^{-r_f(t-1)\cdot(t-1)} \cdot \widetilde{V}(t-1)]$$

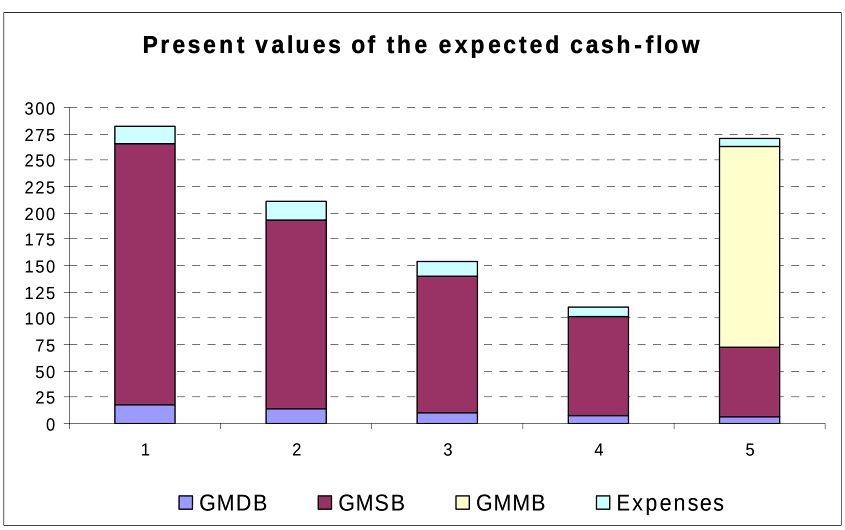
$$- [(\gamma_1 + \gamma_2) + \mu(x, t-1) \cdot z] \cdot e^{-r_f(t)} \cdot (1-\kappa) \cdot B(t-1))$$

where $B(0) = 0$ and $\widetilde{V}(0) = 1000$.

C.11.10 Applying the theory outlined above the following table and figures can be constructed, where GMDB is denotes the benefits with a minimum guaranteed death benefit component, GMSB denotes the benefits with a guaranteed minimum surrender benefit component and GMMB denotes the benefits with a guaranteed minimum maturity benefit component:

Benefits liability before reduction of future premiums	1 139.66
Probability weighted future premiums	175.67
Benefits liability after reduction of future premiums	963.99
- Guaranteed benefits	840.34
- Intrinsic value of extra benefits	73.05
- Time value of guarantees	50.60
Expense liability	67.64
- Future expense loadings	64.41
- Additional liability related to insufficient expense loadings	3.22
Made charges from the benefit fund	80.16
- Future expense loadings	64.41
- Future charges for the cost of the guarantee	15.75
Total best estimate liability	1 031.63





C.11.11 The impact of having a series of cumulative forward start options where annual extra benefits are locked in (annual investment guarantees) can clearly be seen as a rather high time value of guarantees. The effect of removing this by having only a maturity guarantee for instance and by keeping other assumptions unchanged would be observed as a decrease in time value.

C.12 Illustrative example B

C.12.1 The following example illustrates a first insight approach. It is based on illustrative example A that is further simplified in the following way (values are set in parenthesis):

- No premiums are expected to be paid in the future.
- The surrender intensity is constant over time and given by[6] $\gamma_t = 0.3$.
- The present value of expected future guaranteed benefits (862.28) and the present value of related future expense loadings (45.97) are estimated as above.
- The current reserve (applying current valuation basis) equals 1000.
- The calculatory profit/loss fund (91.76) equals current reserve (1000) minus the sum of the present value of expected future guaranteed benefits (862.28) and the present value of expected future expense loadings related to guaranteed benefits (45.97).
- The expected present value of the future expense loadings related to extra benefits (4.89) equals present value of expected future expense loadings related to guaranteed benefits (45.97) divided by present value of expected future guaranteed benefits (862.28) times the calculatory fund (91,76).
- As in illustrative example A it is assumed that historical analysis indicates a 5 percent insufficiency in expense loadings. The additional liability related to insufficient expense loadings (2.54) is thus 5 percent times the sum of the present value of expected future expense loadings related to guaranteed benefits (45.97) and extra benefits (4.89).
- The expected amount of future extra benefits before any firm specific strategies for distributing extra benefits (84.32) equals the calculatory fund (91.76) less the sum of the present value of expected future expense loadings related to extra benefits (4.89) and any additional liability related to insufficient expense loadings (2.54).

[6] Note that we actually assume a slight increase in surrender intensity compared to illustrative example A.

- The expected future extra benefits after firm specific strategies for distributing extra benefits (75.89) equals 90 percent of the expected amount of future extra benefits before any firm specific strategies for distributing extra benefits (84.32).

C.12.2 Applying the principles outlined above the following table can be constructed:

Benefits liability	936.55
- Guaranteed benefits	862.28
- Extra benefits	75.89
Expense liability	53.40
- Related to guaranteed benefits	45.97
- Related to extra benefits	4.89
- Additional liability related to insufficient expense loadings	2.54
Total best estimate liability	991.75

C.12.3 Comparing the illustrative examples A and B one observes that the B example neglects (at least any amount is not transparently tractable) the time value of the options related to the guarantees. Clearly, the stochastic variation of future investment returns in connection with investment guarantees and with-profits contracts creates additional liabilities.

C.12.4 Furthermore, it should be noted that the approach in illustrative example B is quite often artificially bounded above by current reserves held. Circumstances where the total best estimate liability will be in excess of current reserves held are for instances situations where the present value of expected future guaranteed benefits and the present value of related future expense loadings are larger than current reserves held.

C.12.5 Due to its simplicity, illustrative example B nevertheless forms a plausible first insight valuation approach.

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