A Primer for Calculating the Swiss Solvency Test “Cost of Capital” for a Market Value Margin

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1. Introduction

The following document uses simple examples to show different ways how to calculate the (Swiss Solvency Test) SST cost of capital margin (CoCM). For an explanation on the underlying ideas and rational of the cost of capital approach for risk margins, the reader is referred to [1], [2] and [3].

In this version of the document, no references are made to the SST specific approach of aggregating scenarios to the regulatory capital requirement.

To emphasize the underlying ideas, we start with the ‘correct’ calculation, using no simplification. Then we present different ways on how to simplify the calculation for P&C and for life companies.

The examples in this document are purely illustrative and do not correspond to any specific insurance company (short “insurer” in the sequel). We use the term Cost of Capital Margin (CoCM) to refer to the cost of capital approximation of the Market Value Margin (MVM) used in the SST.

It is the responsibility of the insurer to set up the cost of capital margin of identifying the proxy which best relates to future SCR. FOPI considers the assessment of future required regulatory or economic capital an important part of risk management and essential to serve the interests of a insurer’s policyholders and shareholders alike and expects that this or something similar would be done also without an explicit requirement by the supervisor.

Notation

For notation we set t=0 to mean the beginning of year 0. The time interval between t=0 and t=1 is year 0. Analogously, year k means the time interval between t=k and t=(k+1).

The regulatory capital for the one year risk during year k is denoted by SCR(k). In the following we explain in more detail what SCR(k) contains and on which risk it depends.

We denote by P(0,d) the price of a zero coupon bond at t=0 with a duration of d years.

2. Generic Calculation

The CoCM is defined as the cost of future regulatory capital for the whole run-off of a portfolio. Note that by run-off we mean that no new business is taken into account. The insurer is still considered to be a going-concern.

The insurer has to determine or model the market consistent price of the portfolio of liabilities existing at t = 1. This is the fair price for which another party would take over the portfolios of assets and liabilities.

We assume that the insurer taking over the portfolios has to set up future incremental regulatory capital SCR(1), SCR(2) until the portfolios have run-off completely at t=T. The insurer selling the portfolio has to compensate the insurer taking over the portfolio via the CoCM.

The calculations are done on a going-concern basis, meaning that it is assumed that the insurer taking over the portfolio will be a going concern. However it can be assumed for the purpose of calculating the cost of capital margin that no new business is written by the insurer having to set up the margin.

We furthermore assume that there is no additional diversification benefit for the insurer taking over the portfolio. This is done so that even if no insurer can be found to take over the portfolios, the CoCM is sufficient to serve for a run-off of the liabilities.

The Market Value Margin is calculated on a legal entity level. For the purpose of the SST, it is sufficient to determine the CoCM on a portfolio level, i.e. not on a line of business level.

If the insurer has put risk mitigations in place during year 1 or later years, then they reduce financial market, credit and insurance risk, and accordingly the CoCM, according to the mitigants' contractual features, but they might give rise to additional counterparty credit risk.

The insurer calculating the CoCM has to determine the future required regulatory capital SCR(1), SCR(2),…,SCR(T) assuming that no new business is written and that the initial asset portfolio at t = 1 is changed as quickly as possible to a portfolio of assets which replicates optimally the liabilities. This asset portfolio is called the optimal replicating portfolio.

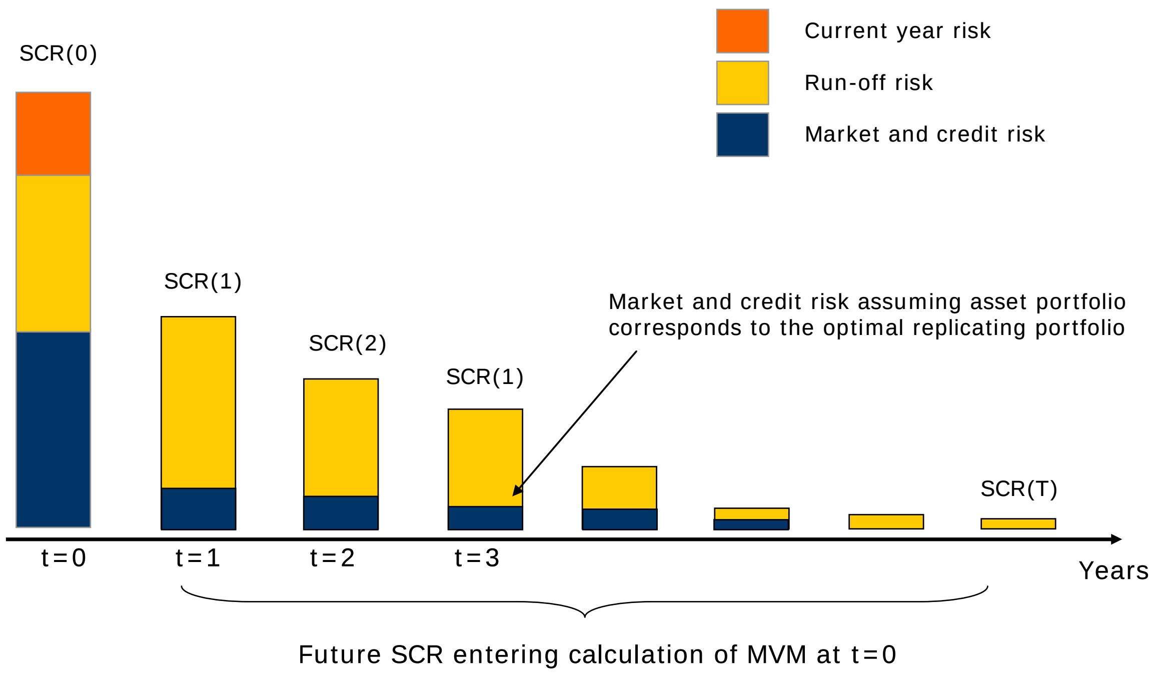
The main task for determining the CoCM is therefore to determine future required regulatory capital associated with the portfolio.

For the sake of simplicity we assume in this document that the asset portfolio can be rebalanced instantaneously to an optimal replicating one.

In a first step the insurer has to determine the optimal replicating asset portfolio. It is composed of traded financial instruments and replicates best the liability cash flows.

The insurer now calculates the regulatory capital necessary for year 1, SCR(1). In contrast to the regulatory capital necessary for time 0 (SCR(0)), SCR(1) does not depend on current year risks (risks of new claims). Hence the remaining insurance risk for SCR(1) is equal to the run-off risk component only. The credit risk component for assets is determined using the credit risk of the optimal replicating portfolio. Financial market risk is determined also using the optimal replicating portfolio. The risks are understood to have a time horizon of 1 year, meaning that for instance financial market risk for SCR(t) corresponds to the financial market risk during year t (e.g. changes in financial market risk factors during year t). Regulatory capital for t=1, SCR(1), and more generally SCR(t), t≥1, depends therefore on the following risks:

- Insurance Risk: Use only run-off risk;
- Financial market Risk: Determine using the optimal replicating portfolio;
- Credit risk: Corresponding to the assumption of an optimal replicating portfolio as investment portfolio and other credit risk than arising from investments, e.g. counterparty credit risk against reinsurers;



In a second step, the above calculation is done for years t=2, 3,…, T, obtaining SCR(1), SCR(2),…, SCR(T). The risk free rate is used to discount SCR(t), t = 1,…,T and calculate the CoCM:

$$CoCM=CoC * (P(0,1) * SCR(1) + \cdots + P(0,T) * SCR(T)).$$

For the SST, the cost of capital CoC is set to 6%.

3. Simplifications

In order to determine SCR of future years in the generic calculation, a full SST for the whole the run-off has to be performed for each of those years. It needs to be acknowledged that the further away the time horizon is, the more uncertain any such calculation becomes (the situation is often figuratively described as the “funnel of uncertainty”). For many companies, it is probably sufficient to approximate full SSTs for future year of the run-off, hence a number of simplifications can be tried.

The cost of capital approach is amenable to a number of simplification and the results under the different simplified schemes are robust.

The main idea of the simplifications is to make the future required regulatory capital SCR(t) at time t dependent on an underlying proxy p(t), which is simpler to determine than SCR(t).

For instance, the underlying proxy could be the best estimate of the run-off portfolio of liabilities at time t, the sum insured (or in L&H: the sum at risk) at time t, or the number of expected claims at time t.

If the underlying proxy p(t) has been determined, then future SCR(t) can be calculated easily. For instance, if the underlying proxy is the best estimate of liabilities BE(t), then

$$SCR(0) \div BE(0) \approx SCR(t) \div BE(t), \text{ hence}$$

$$SCR(t) \approx SCR(0) * BE(t) \div BE(0).$$

Different sub-portfolios might have different proxies. In that case an insurer might project future SCR for the sub-portfolios using different proxies. The so obtained SCR for the sub-portfolios have then to be aggregated to the SCR for the total portfolio, taking into account the dependency structure.

It is the responsibility of the insurers to make plausible their choice of proxies and the simplifications and assumptions chosen for determining the cost of capital.

In the following we look at simplified methods for life and for P&C separately. In the examples which follow we assume that the switch to the optimal replicating portfolio is instantaneous and that credit risk of the replicating portfolio can be neglected.

3.1. P&C Companies

Simplification 1 for P&C companies

We assume that the future SCR (given a replicating portfolio) is proportional to the best estimate of the liabilities during the run-off. This approximation might underestimate the future SCR somewhat since stochastic risk increases (relatively) the smaller the run-off portfolio becomes. Alternatively, this approximation might overestimate the risks since over time, even if a case is not settled, the risk decreases owing to an increase of available information about the remaining cases. In many cases this is however only relevant at the end of the run-off where the SCRs are relatively small and impact negligible.

Step 1: Calculate SCR(0) for t=0, taking into account only insurance run-off risk and financial market and credit risk assuming the optimal replicating portfolio. For P&C business in many cases the optimal replicating portfolio can be taken to be composed of government bonds. If the duration of the portfolio is not too long (which is often the case for P&C liabilities), the expected cash flows can be replicated by government bonds and no financial market or credit risk remains. Then determine run-off risk split into parameter and stochastic risk ($SCR_{run\text{-}off,p}(0)$ and $SCR_{run\text{-}off,s}(0)$, also known as measurement and process risk, respectively).

Step 2: Determine the future best estimate of liabilities of the run-off portfolio: BE(1), BE(2),…,BE(T), prospectively taking the expected inflation into account.

Step 3: Project future $SCR_{run\text{-}off,p}(t)$ by using best estimate of liabilities as a proxy:

$$SCR_{run\text{-}off,p}(t) = SCR_{run\text{-}off,p}(0) * BE(t) \div BE(0)$$

and future $SCR_{run\text{-}off,s}(t)$ be setting

$SCR_{run\text{-}off,s}(t) = SCR_{run\text{-}off,s}(0) * (BE(t) \div BE(0))^{0.5}$. The rational for this proxy is that stochastic risk is diversifiable with the size of the portfolio and – under normality assumptions – follows the square root law. This presupposes that best estimate is a proxy for the number of policies and that the capital requirement is proportional to the standard deviation which would be the case for both Value at Risk and Expected Shortfall for normal distributions.

Step 4: Aggregate $SCR_{run\text{-}off,p}(t)$ and $SCR_{run\text{-}off,s}(t)$ assuming independence either by convolution of the respective density function or by using the square root of squares approximation:

$$SCR_{run\text{-}off}(t) = (SCR_{run\text{-}off,s}(t)^2 + SCR_{run\text{-}off,p}(t)^2)^{0.5}.$$

Note that the insurer has to show that the square root of squares approximation makes sense and has to compare it to a correct approach using distributions for $SCR_{run\text{-}off,s}$ *and* $SCR_{run\text{-}off,p}$.

Example

We consider an example where financial market risk is assumed to be immaterial and the run-off of the insurance portfolio takes until T=5 (end of year 5). Insurance risk is composed of run-off risk emanating from parameter and stochastic risk. The numbers are for illustration purposes only. The discount rate is assumed to be 3% flat (therefore the zero coupon bond prices are $P(0,t)=1/1.03^t$). The cost of capital equals 6%.

Numbers in bold have to be inputted, the other numbers are derived from calculations.

		Year	0	1	2	3	4	5
Proxy								
	Best Estimate of Liabilities		**100**	**70**	**50**	**30**	**10**	**5**
SCR Components								
	$SCR_{run\text{-}off,p}$	parameter risk	**25.00**	17.50	12.50	7.50	2.50	1.25
	$SCR_{run\text{-}off,s}$	stochastic risk	**10.00**	8.37	5.92	3.24	1.02	0.23
Aggregation								
	run-off + stoch	Assume independence	26.93	19.40	13.83	8.17	2.70	1.27
	SCR (total)		26.93	19.40	13.83	8.17	2.70	1.27
	Discounting		**1.00**	**0.97**	**0.94**	**0.92**	**0.89**	**0.86**
	Discounted SCR		26.93	18.83	13.04	7.48	2.40	1.10
	Present Value		42.84					
	Cost of Capital		2.57					

The CoCM so obtained is 2.57.

Simplification 2 for P&C Companies

The second simplification works as above except that the insurance SCR is not split into parameter and stochastic risk. $SCR_{run\text{-}off}$ is considered as a whole and projected forward proportional to the best estimate of liabilities.

Example

The example uses the same assumptions as above. It was assumed that $SCR_{run\text{-}off,p}$ and $SCR_{run\text{-}off,s}$ are independent and $SCR_{run\text{-}off}$ equals

$$SCR_{run\text{-}off}(0) = (SCR_{run\text{-}off,s}(0)^2 + SCR_{run\text{-}off,p}(0)^2)^{0.5}.$$

In practice $SCR_{run\text{-}off}(0)$ is a result which is obtained explicitly when doing the SST calculation, so no additional calculations are required. The results under the second simplified scheme can be seen below.

For this simplified method, the insurer determines the best estimate of the run-off of liabilities. Then it projects the insurance risk component of SCR proportional to the best estimate. If there would be residual financial market and credit risk, these could also be projected proportional and aggregated to the insurance risk component of future SCR.

	Year	0	1	2	3	4	5
Proxy							
	Best Estimate of Liabilities	**100**	**70**	**50**	**30**	**10**	**5**
SCR Component							
	$SCR_{run\text{-}off}$	**26.93**	18.85	13.46	8.08	2.69	1.35
	SCR (total)	26.93	18.85	13.46	8.08	2.69	1.35
	Discounting	**1.00**	**0.97**	**0.94**	**0.92**	**0.89**	**0.86**
	Discounted SCR	26.93	18.30	12.69	7.39	2.39	1.16
	Present Value	41.94					
	Cost of Capital	2.52					

The result under scheme 2 is approximately 2% smaller than under scheme 1 which can be considered to be immaterial.

3.2. Life Companies

Simplification 1 for Life Companies

Companies using the life standard model could use the standard model to do a SST calculation for each year of the run off without too much effort. However, also for life companies, some simplifications are possible.

Step 1: In the first step the portfolio can be split into different risk types, for instance into

- Savings products;
- Mortality Risk products;
- Disability Risk products.

For savings products, use the evolution of the sum insured as a proxy for future SCR.

For mortality products, use the evolution of the future death benefits

For risk products, the present value of future claims can be considered as a reasonable proxy.

For disability products, a proxy might be the number of expected future claims.

If stochastic risk is relevant, then calculate $SCR_{stoch}(0)$ and project as follows:

$$SCR\,(t)_{stoch} = SCR(0)_{stoch} * (BE(t) \div BE(0))^{0.5}.$$

Note that for some products the best estimate of liabilities might be inappropriate, since for instance some risk products have negative reserves. In these cases another proxy has to be found and present value of future claims might be a more appropriate choice.

Financial market risk can be projected proportional to the evolution of the best estimate of liabilities:

$$SCR(t)_{market} = SCR(0)_{market} * (BE(t) \div BE(0)).$$

Step 2: To aggregate the different stand-alone SCR (e.g. $SCR(t)_{market}$, $SCR(t)_{disability}$, etc.), use the dependency assumptions of the standard model.

Step 3: Discount future SCR(t), add up discounted SCR(1) to SCR(T) and multiply by the cost of capital (6%) to arrive at the CoCM.

Example

The example below shows conceptually how the simplified method works for a life company writing savings and risk products. We assume that the liabilities run off until T=6 (end of year 6).

The evolution of the components of SCR can then be determined using as proxies the best estimate and the sum insured and the values for the different components of SCR(0) for t=0. Proxies used:

- Best estimate of liabilities for stochastic risk and financial market risk;
- Best estimate of liabilities for risk products;
- Sum insured for savings products.

Note that for this example we used best estimate of liabilities as a proxy for risk products. This is in many cases (in particular when best estimate would be negative) an inappropriate proxy. The choice of proxy in the example is purely for illustrative purposes.

Aggregation between the different risk components was done as follows:

- SCR between risk and savings products was assumed to be totally negatively dependent and SCRs are added.
- SCR between financial market risk and stochastic risk and SCR of savings and risk combined was assumed to be independent. Then

$$SCR = (SCR_{stoch}^2 + SCR_{markt}^2 + (SCR_{risk} + SCR_{sav})^2)^{0.5}.$$

		Year	0	1	2	3	4	5	6
Proxies									
	Best Estimate	Stochastic Risk	**200**	**150**	**110**	**70**	**40**	**20**	**10**
	Sum Insured	Savings Products	**1000**	**800**	**600**	**400**	**200**	**100**	**0**
	Best Estimate	Risk Products	**100**	**70**	**50**	**40**	**30**	**20**	**10**
	Best Estimate	Market Risk	**200**	**150**	**110**	**70**	**40**	**20**	**10**
SCR Components									
	SCR_{stoch}		**2.00**	1.73	1.48	1.18	0.89	0.63	0.45
	SCR_{sav}		**15.00**	12.00	9.00	6.00	3.00	1.50	0.00
	SCR_{risk}		**5.00**	3.50	2.50	2.00	1.50	1.00	0.50
	SCR_{market}		**10.00**	7.50	5.50	3.50	2.00	1.00	0.50
Aggregation									
	Risk + Saving	assume full dependence	20.00	15.50	11.50	8.00	4.50	2.50	0.50
	+ Stoch	assume independence	20.10	15.60	11.60	8.09	4.59	2.58	0.67
	+ Market	assume independence	22.45	17.31	12.83	8.81	5.00	2.77	0.84
	SCR (total)		22.45	17.31	12.83	8.81	5.00	2.77	0.84
	Discounting		**1.00**	**0.97**	**0.94**	**0.92**	**0.89**	**0.86**	**0.84**
	Discounted SCR		22.45	16.80	12.10	8.06	4.45	2.39	0.70
	Present Value		44.50						
	Cost of Capital		2.67						

The CoCM obtained is 2.67.

Simplification 2 for Life Companies

It is also possible to simplify the calculation method further. For instance by not splitting the insurance risk components into risk, savings and stochastic risk, the evolution of SCR can be made dependent on only one proxy, e.g. the best estimate of liabilities. We assume in the example as above that saving and risk SCR are dependent and stochastic risk is independent. Then

$$SCR_{ins}(0) = ((15 + 5)^2 + 2^2)^{0.5} = 20.1.$$

For this simplified method, the insurer determines the best estimate of the run-off of liabilities. Then it projects the insurance and financial market risk components of SCR proportional to the best estimate. If there would be residual credit risk, it could also be projected proportional and aggregated accordingly.

We use the best estimate of liabilities as a proxy for the evolution of future SCR. The CoCM obtained is 2.52, which is 5% lower than under simplification 1, a difference which is not material.

		Year	0	1	2	3	4	5	6
Proxy									
	Best Estimate		**200**	**150**	**110**	**70**	**40**	**20**	**10**
SCR Components									
	$SCR_{insurance}$		**20.10**	15.08	11.06	7.04	4.02	2.01	1.01
	SCR_{market}		**10.00**	7.50	5.50	3.50	2.00	1.00	0.50
Aggregation									
	+ Market	assume independence	22.45	16.84	12.35	7.86	4.49	2.25	1.12
	SCR (total)		22.45	16.84	12.35	7.86	4.49	2.25	1.12
	Discounting		**1.00**	**0.97**	**0.94**	**0.92**	**0.89**	**0.86**	**0.84**
	Discounted SCR		22.45	16.35	11.64	7.19	3.99	1.94	0.94
	Present Value		42.04						
	Cost of Capital		2.52						

Simplification 3 for Life Companies

This simplification is the same as the example above, but replacing the proportionality factor with the sum insured instead of the best estimate of liabilities.

		Year	0	1	2	3	4	5	6
Proxy									
	Best Estimate		**1000**	**800**	**600**	**400**	**200**	**100**	**0**
SCR Components									
	$SCR_{insurance}$		**20.10**	16.08	12.06	8.04	4.02	2.01	0.00
	SCR_{market}		**10.00**	8.00	6.00	4.00	2.00	1.00	0.00
Aggregation									
	+ Market	assume full dependence	22.45	17.96	13.47	8.98	4.49	2.25	0.00
	SCR (total)		22.45	17.96	13.47	8.98	4.49	2.25	0.00
	Discounting		**1.00**	**0.97**	**0.94**	**0.92**	**0.89**	**0.86**	**0.84**
	Discounted SCR		22.45	17.44	12.70	8.22	3.99	1.94	0.00
	Present Value		44.28						
	Cost of Capital		2.66						

Using this simplification, the CoCM is 2.66, which is about 0.5% lower than under the first simplified scheme.

In this example, the range of values for the CoCM is from 2.52 to 2.67 compared to a best estimate of liabilities of 200 and an SCR of 24. In the field test 2005 of the SST, life CoCM varied between 2% and 8% of best estimate so a variation of 5% in estimating the CoCM is less than 0.5% of the best estimate even for a life company having a large (relative) CoCM. It should however be stressed that this is only an illustrative example and for life portfolios, the duration of the run-off pattern is often longer than in the example above.

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4. References

[1] The Swiss Experience with Market Consistent Technical Provisions - the Cost of Capital Approach, Federal Office of Private Insurance, March 28, 2006

[2] A market cost of capital approach to market value margins, The Chief Risk Officer Forum, 17 March 2006

[3] CEA Document on Cost of Capital, Comité Européen des Assurances, Brussels, 21 April 2006