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THE OPTIMAL EURO CONVERSION RATE IN A STOCHASTIC DYNAMIC GENERAL EQUILIBRIUM MODEL

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Abstract

This paper, using a stochastic dynamic general equilibrium framework, considers how a small open EMU accession country should choose its Euro conversion rate. In this model a monetary union is interpreted as a perfectly credible infinite nominal exchange rate peg, and an algorithm is provided which maps the vector of accession date values of the state and the exogenous variables to a certain size of nominal exchange rate devaluation or revaluation. It is shown that it is not enough to base the decision on exclusively one factor, namely, the real exchange rate misalignment, although this has a primary role in the determination of the optimal conversion rate. Beyond the real exchange rate, the inflation rate, the real and the nominal wage level, the state of the foreign business cycle, as well as foreign price levels and productivity are the most important additional factors necessary for finding the optimal solution.

Contents

1 Introduction

At the beginning of May, 2004, ten countries joined the European Union. It is obligatory for the new member countries to join the Monetary Union, although the deadline for this is not yet specified. Joining the Monetary Union raises several complicated questions of economic policy: policy makers have to decide how and when to meet the Maastricht criteria, they should decide about the date of entering the ERM II exchange rate arrangement, as well as about the corresponding central parity, about the date of joining the Monetary Union, and last but not least, about the Euro conversion rate. This paper aims to contribute to the solution of this last-mentioned problem.

So far, academic literature has not paid attention to this problem. Non-academic economic policy literature focuses almost exclusively on one factor, namely the misalignment of the real exchange rate, namely its deviation from an estimated *equilibrium real exchange rate*.[1] Moreover, it has not provided enough guidelines about how to use the misalignment indices for the determination of the optimal conversion rate. It is not sufficient to base such an important decision on the intuitive wisdom that an overvalued (undervalued) real exchange rate should imply a devaluation (revaluation) of the nominal exchange rate.

This paper performs a welfare analysis of the problem. The Monetary Union is modelled as an infinitely long, perfectly credible exchange rate peg, and a *new open economy macroeconomics* model is used to provide an algorithm to determine how to peg the nominal exchange rate optimally if the accession date values of state and exogenous variables are known.

It is shown that beyond an appropriately defined misalignment index the *past inflation rate* and the level of *real wages* are important state variables worthwhile to take into consideration for the settlement of the conversion rate. Furthermore, the *foreign-business-cycle*, *foreign-price*, *nominal-wage* and *productivity* shocks are the most important exogenous factors necessary for a proper policy decision.

This study demonstrates the importance of a *utility-based* approach and that evaluations based on *ad-hoc* welfare criteria may lead to misleading results. It is shown that the persistence of the inflation process implies that the optimal reaction to a positive past inflation rate is the *devaluation* of the nominal exchange rate. This surprising result is the consequence of the form of the exact social welfare function one can derive from the model: what matters is not the inflation rate itself, but its *quasi-difference* if there is

[1] On different equilibrium real exchange rate concepts see the survey of MacDonald (2000) and Driver and Westaway (2003).

inflation *indexation* in the model. Furthermore, it is shown that the optimal solution is sensitive to the *persistency* parameters of the inflation process.

The paper is structured as follows. *Section 2* presents the model. *Section 3* compares it with other models in the literature. In *section 4* impulse responses are analyzed and characteristics of the optimal conversion rate are discussed. *Section 5* presents the conclusions.

2 The model

For the study of the determination of the optimal conversion rate one has to take into account two important model building issues. On the one hand, the model should be rich enough to capture important characteristics of real-life economic policy problems. On the other hand, it should be simple enough in order to be suitable for an exact welfare analysis.

Thus, instead of using a highly stylized environment the model is based on a rich theoretical framework, and it features nominal and real rigidities, such as sticky prices and wages complemented by implicit indexation, as well as habit formation in consumption.

However, as Woodford (2003, ch. 6) shows, in general equilibrium models some simplifying assumptions are required for the derivation of tractable social welfare functions. Therefore, in this model some restrictions on the relative movements of domestic consumption and exports are imposed, and it is assumed that production inputs, imports and labor, are complements, i.e. firms use Leontieff technology.

2.1 Households

The domestic economy is populated by a continuum of infinitely-lived households. The expected utility function of household j is

$$\sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_1 \left[u(H_t(j)) - v(l_t(j)) \right], \tag{1}$$

for all $j \in [0,1]$. $H_t(j) = c_t(j) - hc_{t-1}(j)$, where $c_t(j)$, $c_{t-1}(j)$ denote the consumption of household j at date t and $t-1$, parameter $h \in [0,1)$ measures the strength of habit formation,[2] and $l_t(j)$ is the labor supply of household

[2] The consumption habit is defined by past individual consumption and not by past aggregate consumption. The reason is that if the consumption habit is related to aggregate consumption, then there is an extra externality in the model, which would make the welfare analysis of the paper more complicated.

j. Furthermore $u(H) = H^{1-\sigma}/(1-\sigma)$, and $v(l) = l^{1+\varphi}/(1+\varphi)$, σ, $\varphi > 0$, and $0 < \beta < 1$.

The intertemporal budget constraint of a given household can be written in the form

$$P_t c_t(j) + P_t^B B_t(j) = \zeta_t(j) B_{t-1}(j) + \left(1 - \tau_t^W\right) W_t(j) l_t(j) + T_t(j), \quad (2)$$

where P_t is the consumer price index, $B_t(j)$ is the household's nominal portfolio at the beginning of time t, P_t^B is its price, $\zeta_t(j)$ is its stochastic payoff. $W_t(j)$ is the nominal wage paid to household j, τ_t^w represents labor market taxes and transfers, and $T_t(j)$ is a lump-sum tax/transfer levied/paid by the government. Households supply differentiated labor, hence the wage paid to individual households can be different. On the other hand, it is assumed that the asset markets are complete, and it is possible to eliminate the risk of heterogeneous labor supply and labor income.[3] As a consequence, households have uniform income and consumption, i.e. $c_t(j)=c_t$, and they have the same portfolio, i.e. $B_t(j) = B_t$, for all j and t.

The optimization problem of the households is the following: they maximize the objective function (1) subject to budget constraint (2), non-negativity constraints on consumption, and no-Ponzi schemes. The assumption of complete asset markets implies that the intertemporal allocation of consumption is determined by the following condition in all states of the world:

$$\beta \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} = D_{t,t+1}, \quad (3)$$

where Λ_t is the marginal utility of consumption,

$$\Lambda_t = (c_t - hc_{t-1})^{-\sigma} - \beta h \mathrm{E}_t \left[c_{t+1} - hc_t\right]^{-\sigma}, \quad (4)$$

and $D_{t,t+1}$ is the stochastic discount factor, which satisfies the condition

$$P_t^B = \mathrm{E}_t \left[D_{t,t+1} \zeta_t\right].$$

Since it is assumed that markets of international assets are also complete, foreign equivalent of equation (3) is also held,

$$\beta \frac{\Lambda_{t+1}^* e_t P_t^{F*}}{\Lambda_t^* e_{t+1} P_{t+1}^{F*}} = D_{t,t+1}, \quad (5)$$

where Λ_t^* is the marginal utility of foreign households, P_t^{F*} is the foreign consumer price index in foreign currency terms, and e_t is the nominal exchange

[3] It is assumed that the government's budget is balanced. The labor tax/transfer policy represented by τ_t^w is compensated by the non-distortive T_t lump-sum tax/transfer.

rate. Combining equations (3) and (5), and applying recursive substitutions gives:

$$\frac{\Lambda_t q_t^d P_t^{F*}}{\Lambda_t^*} = \iota, \tag{6}$$

where ι is a constant, which depends on initial conditions, and

$$q_t^d = \frac{e_t}{p_t}.$$

Since $P_t^{F*} q_t^d$ is the real exchange rate, q_t^d is called the *domestic component of the real exchange rate.*

There is monopolistic competition in the labor market: As mentioned, labor is differentiated, hence nominal wages can be different, and it is assumed that $W_t(j)$ is set by household j. This implies that the demand for labor supplied by household j is given by

$$l_t(j) = \left(\frac{W_t}{W_t(j)}\right)^{\theta_w} l_t, \tag{7}$$

where the aggregate wage index W_t is defined by

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta_w}\, dj\right)^{\frac{1}{1-\theta_w}}.$$

It is assumed that there is sticky wage setting in the model, as in the paper of Erceg et al. (2000). Similarly to Calvo (1983), every individual household at a given date changes its wage in a rational, optimizing forward-looking manner with probability $1-\gamma_w$. All those households, which do not behave like this at the given date follow a rule of thumb, as in the models of Christiano et al. (2001) and Smets and Wouters (2003), and they update their wages according to the past inflation rate. Each household which sets its price optimally takes into account the above mentioned characteristics of the wage setting process, and the form of the labor demand function represented by equation (7). *Appendix B.2* shows that these conditions imply that wage formation is determined by the following equation:

$$\pi_t^w - \vartheta_w \pi_{t-1} = \beta \mathrm{E}_t \left[\pi_{t+1}^w - \vartheta_w \pi_t\right] + \xi_w \left[\widetilde{mrs}_t - \widetilde{w}_t\right] + \tilde{\upsilon}_t^w, \tag{8}$$

where tilde denotes the percentage deviation of the variables from their steady-state values, and

$$\xi_w = \frac{(1-\gamma_w)(1-\beta\gamma_w)}{\gamma_w(1+\varphi\theta_w)}, \tag{9}$$

where $\pi_t^w = \widetilde{W}_t - \widetilde{W}_{t-1}$ is nominal wage inflation, $\pi_t = \widetilde{P}_t - \widetilde{P}_{t-1}$, is CPI inflation, $\vartheta_w \in [0,1]$ measures the degree of implicit indexation applied by those who follow the rule of thumb, and the average marginal rate of substitution between consumption and labor is defined by

$$mrs_t = l_t^{\varphi} \Lambda_t^{-1}. \tag{10}$$

The exogenous shock $\tilde{\upsilon}_t^w$ is given by $\tilde{\upsilon}_t^w = \xi_w \tilde{\mu}_t^w + \tilde{\varepsilon}_t = \xi_w \tau^w \left(1 - \tau^w\right)^{-1} \tilde{\tau}_t^w + \tilde{\varepsilon}_t$, where τ^w is the steady state value of $\tilde{\tau}_t^w$, and the interpretation of $\tilde{\varepsilon}_t$ is the same as in footnote 14 of the paper of Clarida et al. (1999): this represents some kind of systematic error in wage formation.

2.2 Production

Production has a hierarchical structure: at the first stage, import goods and labor are transformed into differentiated intermediate goods, at the second stage, a homogenous final good is produced by the differentiated goods.

Final good y_t is produced in a competitive market by a constant-returns-to-scale technology from a continuum of differentiated intermediate goods $y_t(i)$, $i \in [0,1]$. The technology is represented by the following CES production function:

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$. As a consequence, price P_t is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}},$$

where $P_t(i)$ denotes the prices of differentiated goods $y_t(i)$, and the demand for $y_t(i)$ is determined by

$$y_t(i) = \left(\frac{P_t}{P_t(i)} \right)^{\theta} y_t. \tag{11}$$

The continuum of goods $y_t(i)$ is produced in a monopolistically competitive market. Each $y_t(i)$ is made by an individual firm, and they apply the same technology. Firm i uses a decreasing-returns-to-scale technology, [4]

[4] Decreasing returns to scale is not a common assumption in macroeconomics. But, as is shown in Woodford (2003, ch. 5), provided the technology of the firms exhibits constant returns to scale, physical capital is firm-specific, and the adjustment cost of investment is high, then the presence of sticky prices implies that the firms' behavior becomes similar to the behavior induced by a decreasing-returns-to-scale technology without capital. In addition, it can be assumed that capital is fixed and normalized 1. This can be justified on the basis that at business cycle frequencies capital is uncorrelated with output.

which is given by

$$y_t(i) = A_t z_t(i)^{1-\alpha},$$

where $0 < \alpha < 1$, A_t is a uniform exogenous productivity factor of the industry, and $z_t(i)$ denotes the firm i's utilization of composite good z_t,

$$z_t(i) = \min\left[a_l^{-1} l_t(i), a_m^{-1} m_t(i)\right],$$

where $l_t(i)$ is the firm's utilization of composite labor l_t defined as

$$l_t = \left(\int_0^1 l_t(j)^{\frac{\theta_w - 1}{\theta_w}} dj\right)^{\frac{\theta_w}{\theta_w - 1}},$$

where $\theta_w > 1$. $m_t(i)$ is the utilization of imported good m_t, and a_l, a_m are given parameters.[5] The price of z_t is determined by

$$W_t^z = a_l W_t + a_m e_t P_t^{m*},$$

where P_t^{m*} is the foreign currency price of the imported good.

The assumptions on the production process imply that $y_t(i)/A_t = z_t^{1-\alpha} = (l_t(i)/a_l)^{1-\alpha} = (m_t(i)/a_m)^{1-\alpha}$. Thus, the demand for labor and import of firm i is determined by

$$l_t(i)^{1-\alpha} = a\frac{y_t(i)}{A_t}, \qquad m_t(i)^{1-\alpha} = (1-a)\frac{y_t(i)}{A_t}, \tag{12}$$

where $a = a_l^{1-\alpha}$ and $(1-a) = a_m^{1-\alpha}$.

It is assumed that prices are sticky: as in the model of Calvo (1983), each firm at a given date changes its price in a rational, optimizing, forward-looking way with probability $1-\gamma$. Those firms which do not optimize at the given date follow a rule of thumb, as in the models of Christiano et al. (2001) and Smets and Wouters (2003), and update their prices according to the past inflation rate. The optimizing firms take into account the above described characteristics of the price setting process, and the form of the demand function represented by equation (11). These conditions imply that the inflation rate is determined by the following equation:

$$\pi_t - \vartheta\pi_{t-1} = \beta \mathrm{E}_t\left[\pi_{t+1} - \vartheta\pi_t\right] + \xi\widetilde{mc}_t + \tilde{\upsilon}_t, \tag{13}$$

where

$$\xi = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma\left[1 + \theta\alpha(1-\alpha)^{-1}\right]}, \tag{14}$$

and $\vartheta \in [0,1]$ is the degree of implicit indexation, and mc_t is the average real marginal cost. The interpretation of the stochastic shock $\tilde{\upsilon}_t$ is similar to that of $\tilde{\upsilon}_t^w$.

[5] Thus, I apply the approach of McCallum and Nelson (2001), Smets and Wouters (2002) and Laxton and Pesenti (2003), who consider imports as a production input.

2.3 Equilibrium conditions

The equilibrium conditions of the goods and labor market are

$$y_t = c_t + x_t, \tag{15}$$

$$l_t = \int_0^1 l_t(i)\, di. \tag{16}$$

2.4 The log-linearized model

This section summarizes the log-linearized equations determining trajectories of the endogenous variables for given initial conditions and paths of the exogenous variables. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

Combination of the log-linearized version of equations (4) and (6) provides the following formula for domestic consumption:

$$\begin{aligned} &\left(1+\beta h^2\right)\tilde{c}_t - \beta h \mathrm{E}_t\left[\tilde{c}_{t+1}\right] - h\tilde{c}_{t-1} \qquad (17) \\ &= \left(1+\beta h^2\right)\tilde{c}_t^* - \beta h \mathrm{E}_t\left[\tilde{c}_{t+1}^*\right] - h\tilde{c}_{t-1}^* + \frac{(1-h)(1-\beta h)}{\sigma}\left(\tilde{q}_t^d + \widetilde{P}_t^{F*}\right). \end{aligned}$$

Foreign behavior is not modelled explicitly, it is just assumed that the following ad hoc formula, similar to the consumption equation, determines the demand for exports:

$$\left(1+\beta h^2\right)\tilde{x}_t - \beta h \mathrm{E}_t\left[\tilde{x}_{t+1}\right] - h\tilde{x}_{t-1} = \eta\tilde{q}_t^d + \tilde{x}_t^*, \tag{18}$$

where $0 < \eta$, shock x_t^* represents the exogenous component of exports demand.

Let us log-linearize the demand functions, then

$$(1-\alpha)\tilde{l}_t(i) = (1-\alpha)\widetilde{m}_t(i) = \tilde{y}_t(i) - \widetilde{A}_t.$$

This implies that

$$(1-\alpha)\tilde{l}_t = (1-\alpha)\widetilde{m}_t = \tilde{y}_t - \widetilde{A}_t$$

since log-linearization neglects second and higher order approximation error terms.[6] The demand for labor can be derived from the previous expression,

$$(1-\alpha)\tilde{l}_t = a\tilde{c}_t + (1-a)\tilde{x}_t - \widetilde{A}_t, \tag{19}$$

[6] If a variable is defined in the following manner: $\mathfrak{z} = \int_0^1 \mathfrak{z}(i)\, di$ then its log-linear approximation yields $\tilde{\mathfrak{z}} = \int_0^1 \tilde{\mathfrak{z}}(i)\, di + o^2$, where o^2 denotes those second and higher order errors, which were neglected in the approximation process.

where it is used that $c/(c+x)=a$.

Substituting the log-linearized real marginal cost into equation (13) yields the price setting equation

$$\begin{aligned}\pi_t-\vartheta\pi_{t-1} &= \beta \mathrm{E}_t\left[\pi_{t+1}-\vartheta\pi_t\right]+\xi\frac{\alpha}{1-\alpha}\left[a\tilde{c}_t+(1-a)\tilde{x}_t\right] \\ &+ \xi\left[a_l\frac{W}{W^z}\widetilde{w}_t+a_m\frac{eP^{m*}}{W^z}\left(\tilde{q}_t^d+\widetilde{P}_t^{m*}\right)-\frac{\widetilde{A}_t}{1-\alpha}\right]+\tilde{\upsilon}_t.\end{aligned} \tag{20}$$

If one combines the log-linearized version of equations (8) and (10), then one obtains the following wage setting equation:

$$\begin{aligned}&\pi_t^w-\vartheta_w\pi_{t-1}=\beta\mathrm{E}_t\left[\pi_{t+1}^w-\vartheta_w\pi_t\right] \\ &+\xi_w\left\{\varphi\tilde{l}_t+\frac{\sigma\left[(1+\beta h^2)\,\tilde{c}_t-\beta h\mathrm{E}_t\left[\tilde{c}_{t+1}\right]-h\tilde{c}_{t-1}\right]}{(1-h)(1-\beta h)}-\widetilde{w}_t\right\}+\tilde{\upsilon}_t^w.\end{aligned} \tag{21}$$

Finally, two identities close the system:

$$\pi_t^w = \widetilde{w}_t-\widetilde{w}_{t-1}+\pi_t, \tag{22}$$

$$\pi_t = \tilde{q}_{t-1}^d-\tilde{q}_t^d+d\tilde{e}_t, \tag{23}$$

where $d\tilde{e}_t=\tilde{e}_t-\tilde{e}_{t-1}$.

The seven-equation system of equations (17)−(23) determines the paths of the following seven endogenous variables: $\tilde{q}_t^d$, π_t, π_t^w, $\tilde{c}_t$, $\tilde{x}_t$, $\tilde{l}_t$, and $\widetilde{w}_t$. The stochastic shocks of the model are $\tilde{c}_t^*$, $\tilde{x}_t^*$, $\widetilde{P}_t^{F*}$, $\widetilde{P}_t^{m*}$, $\tilde{\upsilon}_t$, $\tilde{\upsilon}_t^w$, and $\widetilde{A}_t$.

2.5 The social welfare function and the consumption gap

Following Woodford (2003, ch. 6), *Appendix B.2* demonstrates that the second-order approximation of the model-based social welfare function yields a quadratic formula of the form

$$\begin{aligned}&-\sum_{t=1}^{\infty}\beta^{t-1}\mathrm{E}_1\left[\lambda_c\left(\hat{c}_t-\delta\hat{c}_{t-1}\right)^2\right] \\ &-\sum_{t=1}^{\infty}\beta^{t-1}\mathrm{E}_1\left[\lambda_\pi(\pi_t-\vartheta\pi_{t-1})^2+\lambda_w(\pi_t^w-\vartheta_w\pi_{t-1})^2\right],\end{aligned} \tag{24}$$

where coefficients λ_c, λ_π, λ_w are functions of parameters of the model, and $0<\delta\leq h$ is a function of the parameter h.

I refer to the variable $\hat{c}_t$ as the *consumption gap*. The appearance of the lag of the consumption gap in the objective function (24) is due to habit formation. Variable $\hat{c}_t$ is defined as the percentage deviation of actual consumption from an appropriately defined *welfare reference level* of consumption, i.e.

$$\hat{c}_t = \tilde{c}_t - \tilde{c}_t^{wr}, \tag{25}$$

where

$$\begin{aligned}&\left[\left(1+\beta h^2\right)\bar{\sigma} + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right]\tilde{c}_t^{wr} - \beta h\bar{\sigma}\mathrm{E}_t\left[\tilde{c}_{t+1}^{wr}\right] - h\bar{\sigma}\tilde{c}_{t-1}^{wr} \\ &= \frac{1+\varphi}{1-\alpha}\left[\widetilde{A}_t + (1-a)\left(\tilde{c}_t - \tilde{x}_t\right)\right],\end{aligned} \tag{26}$$

and $\bar{\sigma} = \sigma[(1-h)(1-\beta h)]^{-1}$.

In most models the welfare reference level corresponds to the flexible price and wage version of the model. In this model the welfare reference level is different, since there is an externality, which is the consequence of openness.[7] In the closed economy version of the model $\tilde{c}^{wr}$ would depend only on productivity shock $\widetilde{A}_t$, expression $(1-a)\left(\tilde{c}_t - \tilde{x}_t\right)$ is the outcome of openness.

Using definitions (25) and (26) one can obtain the process determining the consumption gap:

$$\begin{aligned}&\left[\left(1+\beta h^2\right)\bar{\sigma} + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right]\hat{c}_t - \beta h\bar{\sigma}\mathrm{E}_t\left[\hat{c}_{t+1}\right] - h\bar{\sigma}\hat{c}_{t-1} \\ &= \left[\left(1+\beta h^2\right)\bar{\sigma} + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right]\tilde{c}_t - \beta h\bar{\sigma}\mathrm{E}_t\left[\tilde{c}_{t+1}\right] - h\bar{\sigma}\tilde{c}_{t-1} \\ &-\frac{1+\varphi}{1-\alpha}\left[\widetilde{A}_t + (1-a)\left(\tilde{c}_t - \tilde{x}_t\right)\right].\end{aligned} \tag{27}$$

In usual ad-hoc objectives functions of monetary policy there is a CPI inflation rate term. However, the objective function (24) contains the quasi-difference of CPI inflation $\pi_t - \vartheta\pi_{t-1}$, and the difference of wage and lagged CPI inflation $\pi_t^w - \vartheta_w\pi_{t-1}$. Since expression (24) is derived from the model, the above terms represent certain welfare-decreasing distortions of the model. It is assumed that price and wage setting is asynchronized, hence the CPI and wage inflation result in not only a change of the aggregate price and wage indices, but inefficient relative price and wage movements as well. Since the magnitude of relative price and wage distortions depend on the size of implicit indexation, the terms $\vartheta\pi_{t-1}$ and $\vartheta_w\pi_{t-1}^w$ have to appear in the objective function.

[7] Corsetti and Pesenti (2001a) and (2001b) discuss in detail how different actions of domestic economic policy influence foreign welfare.

2.6 Model solution and parameterization

Uhlig's (1999) implementation of the *undetermined coefficients* method is used to derive the resolution of the log-linear model. The numerical results are generated by the aforementioned author's MATLAB algorithm.

The values of the basic parameters in the benchmark economy are given in *Table 1.* The value of β is taken from King and Rebello (1999). Parameters σ and h are taken from Christiano et al. (2001).The value α is chosen to ensure that labor's share in GDP is approximately 50 per cent. In order to get an appropriate approximation of the social welfare function, it is necessary to ensure that difference of consumption and exports, i.e. $\tilde{c}_t - \tilde{x}_t$ depends only on stochastic shocks and initial conditions. This requirement is fulfilled if the parameters of equations (17) and (18) satisfy the condition $\eta = (1-h)(1-\beta h)/\sigma$; details are presented in *Appendix B.2.* The values of φ, θ_w, γ_w, ϑ_w are Euro area estimates, taken from the paper of Smets and Wouters (2003). Since in that paper the pricing equation is estimated under the assumption of constant returns to scale, I take the values of γ_s and ϑ_s from the study of Galí et al. (2001), which also contains Euro area estimates In that study they interpret inflation persistency differently from the approach I use. They use the model of Galí and Gertler (2000), and assume that each firm updates its price in a given period by probability $1-\gamma_G$. Hence, according to the law of large numbers in a given period $1-\gamma_G$ fraction of the firms change their prices. But only $1-\vartheta_G$ fraction of the price setters choose their prices in an optimal forward-looking manner, the rest update their prices according to the past inflation rate. If $\beta = 1$, then the approach I use and the one used by Galí and Gertler coincides, if $\vartheta = \vartheta_G/\gamma_G$. Although in our case $\beta \neq 1$, as an approximation I used the above mentioned formula to determine the value of ϑ_G.

The above parameter values are used in the benchmark simulations. However, an alternative version is considered, where the persistency parameters of the CPI and wage inflation processes are much higher, i.e. $\vartheta = \vartheta_w = 0.9$. This sensitivity analysis is motivated by the study of Hornok and Jakab (2003). They used a Hungarian, relatively short, data set and found much more persistent inflation than Galí and Gertler (2000), Galí et al. (2001), and Smets and Wouters (2003) estimated using US and European data. It is not clear whether this discrepancy is due to the confused expectations induced by the 2001 exchange-rate regime switch in Hungary, or to more rigid price setting behavior. If the first hypothesis is true, then some near-rational expectation models should explain the phenomenon, e.g., adaptive learning models, see Evans and Honkapohja (2001). But in this case, in the long run the price setting behavior would converge to the practice of developed countries, and low persistency parameters would appropriately describe the inflation process. On the other hand, if the price setting practice in Hungary is significantly different from the practice of developed countries, then inflation would remain highly persistent even in the long run. To select the right explanation needs further research and more data.

Table 1
Parameter values of the benchmark economy

Parameter	
Name	Value
β	0.984
σ	1.000
h	0.630
φ	0.755
η	0.141
α	0.100
θ	6.000
θ_w	3.000
γ	0.787
ϑ	0.365
γ_w	0.763
ϑ_w	0.656

3 Comparison with other small open economy models

3.1 Derivation of the social welfare function

An important advantage of utility-based general equilibrium models is that they do not need to rely on ad-hoc social welfare criteria. It is possible to derive an exact social welfare function from the model itself, and this makes a rigorous welfare analysis possible. However, this remains only a possibility if a model is technically intractable. That is why the finding of Woodford (2003, ch. 6) is important, in that he derived a tractable approximation of the social welfare function in a closed economy framework under relatively mild assumptions.

In a closed economy model it is enough to assume that the steady-state allocation satisfies a certain social welfare criterion. This assumption eliminates first-order terms of the approximation of the social welfare function, which is necessary since the presence of these terms makes optimization results inaccurate.

In small open economy models one needs further assumption to ensure the above requirement. E.g. Galí and Monacelli (2002) impose restrictions both on the intratemporal and intertemporal substitution parameter in such a way that international trade is balanced at all times.

My restrictions are different, but they are also related to the behavior of the trade balance. To be more specific, it is assumed that the parameters of equations (17) and (18) satisfy

$$\eta = \frac{(1-h)(1-\beta h)}{\sigma}.$$

This condition ensures that the difference $\tilde{c}_t - \tilde{x}_t$ depends only on stochastic shocks and initial conditions, i.e. it is independent of endogenous variables and economic policy. *Appendix B.2* shows that the above condition and the Leontieff form of the technology guarantee the elimination of first-order terms.[8]

An interesting feature of the model of Galí and Monacelli is that domestic inflation and not CPI inflation is the proper welfare measure. In my model there is no distinction between the two, since domestic consumption goods are home made, and imports are used for production.

3.2 Stabilization of the consumption gap and inflation

It is a well known feature of closed economy New Keynesian models that if wages are flexible and there are no cost-push shocks, then it is possible for there to be the simultaneous stabilization of the relevant welfare measures: the output gap and inflation. See, e.g., Clarida et al. (1999), Goodfriend and King (1997), Galí (2002), and Woodford (2003, ch. 7).

Galí and Monacelli (2002) show that if some certain conditions are satisfied, then even in small open economies the simultaneous stabilization of the output gap and the appropriate measure of inflation is possible.

Let us study whether similar assumptions make simultaneous stabilization possible in my model. Recall that in this model the relevant welfare measure is not the output gap, but a similar concept, the consumption gap. Let us assume that wages are flexible ($1/\xi_w = 0$), and there are no cost push shocks $\tilde{\upsilon}_t = \tilde{\upsilon}_t^w = 0$. Furthermore, for the sake of simpler comparison, suppose $h = 0$, $\alpha = 0$, and $\tilde{x}_t^* = \widetilde{P}_t^{x*} = \widetilde{P}_t^{m*} = 0$.

[8] It could be an interesting further research topic to relax the above restrictive assumptions, e.g., by the application of the methods described in Benigno and Woodford (2003) or Schmitt-Grohé and Uribe (2004).

If wages are flexible, then combining equations (19) and (21) yields

$$(\sigma + a\varphi)\tilde{c}_t + (1-a)\varphi\tilde{x}_t - \varphi\widetilde{A}_t = \widetilde{w}_t.$$

Substituting this into equation (20) yields the following price setting equation:

$$\begin{aligned}\pi_t - \vartheta\pi_{t-1} &= \beta \mathrm{E}_t\left[\pi_{t+1} - \vartheta\pi_t\right] + \xi a \frac{W}{W^z}(\sigma + a\varphi)\tilde{c}_t + \xi a \frac{W}{W^z}(1-a)\varphi\tilde{x}_t \\ &+ \xi(1-a)\frac{eP^{m*}}{W^z}\tilde{q}_t^d - \left(1 + a\frac{W}{W^z}\varphi\right)\widetilde{A}_t.\end{aligned}$$

Replace consumption and exports: since $h = 0$ and $\alpha = 0$, equation (27) implies that

$$\tilde{c}_t = \nu_c\hat{c}_t + \nu\widetilde{A}_t - (1-a)\nu\tilde{x}_t,$$

where $\nu_c = [\sigma(1-a) + \varphi]/[\sigma(1-a) + \varphi - (1+\varphi)\alpha]$ and $\nu = (1+\varphi)/[\sigma(1-a) + \varphi - (1+\varphi)\alpha]$. Using the above expression and equation (18) the price setting equation can be expressed as

$$\begin{aligned}&\pi_t - \vartheta\pi_{t-1} = \beta \mathrm{E}_t\left[\pi_{t+1} - \vartheta\pi_t\right] + \xi a \frac{W}{W^z}(\sigma + a\varphi)\nu_c\hat{c}_t \\ &+\xi\left\{(1-a)\frac{eP^{m*}}{W^z} + a\frac{W}{W^z}\eta\left[(1-a)\varphi - (\sigma + a\varphi)(1-a)\nu\right]\right\}\tilde{q}_t^d \\ &-\left(1 + a\frac{W}{W^z}\varphi - (\sigma + a\varphi)\nu\right)\widetilde{A}_t.\end{aligned}$$

Obviously, in this case it is impossible to stabilize simultaneously the consumption gap and inflation, since in the price setting equation, beyond the consumption gap, there is another endogenous variable $\tilde{q}_t^d$, and it is easy to show that the coefficient of the productivity shock is non-zero. Simultaneous stabilization is possible only in the closed economy version of this model, i.e. when $a = 1$. In this case $W = W^z$, $\nu_c = 1$, and $\nu = (1+\varphi)/(\sigma+\varphi)$, thus the price setting equation becomes the standard New Keynesian closed economy Phillips curve,

$$\pi_t - \vartheta\pi_{t-1} = \beta \mathrm{E}_t\left[\pi_{t+1} - \vartheta\pi_t\right] + \xi(\sigma + \varphi)\hat{c}_t.$$

The model of Monacelli (2003), which can be considered as a generalization of the model of Galí and Monacelli, has the same property, i.e. simultaneous stabilization is impossible. Monacelli in his generalized model relaxes the assumption of perfect import price pass-through. Imperfect pass-through implies his impossibility result. In my model import price pass-through is perfect. But since the imported goods are not used for consumption, the pass-through between the nominal exchange rate and CPI becomes imperfect, as in the model of Monacelli. Thus, it comes as no surprise that in my model simultaneous stabilization is also impossible.

3.3 Fixed versus flexible exchange rate

As mentioned in the *Introduction*, the new member states do not have the option of remaining outside the Monetary Union. From this point of view it is worthless considering whether it is better to maintain an independent exchange rate policy. However, it may still be interesting to briefly discuss this problem.

Although some models with pricing to market and local currency pricing support fixed exchange rate regimes, the majority of the NOEM literature assert that flexible regimes are optimal, see Obstfeld (2001, 2002). The basic argument is the following. Nominal rigidities prevent the economy from optimal adjustment. On the other hand, in the presence of nominal rigidities nominal-exchange-rate policy can influence real-exchange movements. As a consequence, an appropriate exchange rate policy can facilitate the necessary adjustments of real economic variables.[9]

But all the above arguments are based on the assumption that the monetary authority can perfectly control the nominal exchange rate at all times. However, there is a literature relatig to the volatility of nominal exchange rates. As Lyons (2001) discusses, there are two important unresolved puzzles in exchange rate economics: the *determination puzzle*, and the *excess volatility puzzle*. The determination puzzle is related to the fact that exchange rate movements are virtually unrelated to the best measures of fundamentals. The excess volatility puzzle is related to the phenomenon that exchange rates are excessively volatile relative to the best measures of fundamentals. These issues represent especially important problems in emerging market countries, as documented by Calvo and Reinhart (2002). Thus, perfect exchange rate stability supported institutionally by the Monetary Union can be attractive for the new accession countries, and can improve social welfare.

4 The optimal conversion rate

4.1 Analysis of the impulse responses

Before discussing the properties of the optimal solution, it is worthwhile analyzing the characteristics of the model by its impulse-response functions. First, the effects of 1 percent initial (date $t = 0$) deviations of the state variables are discussed. Then the effects of exogenous shocks are studied. For all variables two set of simulations will be performed, one for low persistency

[9]Moreover, in the model of Galí and Monacelli (2002) the increased volatility of CPI inflation induced by a flexible exchange rate is not harmful for the welfare since domestic inflation is the relevant welfare measure.

of inflation ($\vartheta = \vartheta_w = 0.365$), and one for high persistency ($\vartheta = \vartheta_w = 0.9$). The interpretation of one time period in the model is one quarter. The upper four panels of the figures belong to the low persistency case, the lower panels to the high persistency version. Figures display the price and wage inflation rate by annualized terms.

State variables

Figure 1 displays the impulse responses induced by 1 percent annual inflation rate at date 0. The inflation rate returns to its steady-state value in three quarters if the persistency is small, and in one year if it is high, but in this latter case undershooting is significant. Since the fixed nominal exchange rate does not move together with inflation, the domestic component of the real exchange rate appreciates, which results in recession: both the consumption gap and labor utilization decline. If the persistence of inflation is high, then the decline is much stronger. Since it is assumed that $\eta = (1-h)(1-\beta h)/\sigma$, equations (17) and (18) imply that $\tilde{c}_t$ and $\tilde{x}_t$ moves together perfectly. Thus, equation (26) implies that $\tilde{c}^{wr}$ is constant, and formula (27) guarantees that the consumption gap moves together with consumption and exports as well.

Figure 2 displays impulse responses corresponding to consumption. As equation (17) reveals, $\tilde{c}_t$ gradually converges to its steady-state value. The path of labor utilization is similar. On the other hand, the real wage level hardly exhibits any reaction. This is possible, because if wages are sticky, these two variables do not necessarily move together. The reaction of price and wage inflation, and the domestic component of the real exchange rate is negligible compared to that of the consumption gap and labor. If inflation persistence increases, then the size of the movements do not change significantly, but the shape of the trajectories become modified: after converging back to the steady-state values, the variables undershoot, and keep moving cyclically with decreasing amplitude. The effects of a 1 percent initial deviation of exports are similar to that of consumption, hence its graphical analysis is skipped.

Figure 3 plots the impulse responses corresponding to 1 percent initial deviation of the real wage level. This level returns slowly, in more than five years, to its steady-state value. This has two effects: on the one hand, it decreases wage inflation, while, on the other, it increases CPI inflation, though this latter effect is much weaker. In addition, it has a contraction effect both on consumption and labor, but this effect is also weak. Consumption and exports perfectly moves together with the consumption gap again.

It is worthwhile analyzing the effects of the domestic component of the real exchange rate $\tilde{q}^d_0$ and the nominal-exchange-rate depreciation $d\tilde{e}_1$ together. If one investigates the system of equations (17) – (22), and (27), then it becomes apparent that neither $\tilde{q}^d_{t-1}$ nor $d\tilde{e}_t$ appear in it. On the other hand, in identity (23) both appear, and their coefficients are the same in absolute value. This implies that if the domestic component of the real exchange rate is undervalued by 1 percent at date $t = 0$, then it has exactly the same effect as that of a 1 percent depreciation of the nominal exchange rate at date $t = 1$.

Figure 4 shows the impulse responses corresponding to a 1 percent depreciation of the nominal exchange rate at date $t = 1$. It has significant positive effect on the consumption gap, price and wage inflation, and labor utilization. They return to their steady-state values in nearly five years. Since the coefficients of the real exchange rate in equations (17) and (18) are the same, consumption and export move together again. Thus, equation (27) implies that the consumption gap also moves with them.

Exogenous shocks

Now the effects of exogenous stochastic shocks will be investigated. Using the estimation results of Ireland (2004) it is assumed that the autoregressive parameters of the shocks are equal to 0.95.

Let us start with the foreign-business-cycle shock $\tilde{c}^*_t$: *Figure 5* plots the corresponding impulse responses. This shock has a significant impact on the consumption gap, consumption and labor. It increases the real wage level, and consequently, wage inflation as well. The reaction of price inflation is much weaker. The effects of the exports-demand shock $\tilde{x}^*_t$ and the foreign-CPI shock $\widetilde{P}^{F*}_t$ are similar. Although, the increase in wage inflation is weaker in both cases, and the increase in labor utilization is stronger when $\tilde{x}^*_t$ is considered.

The import-price shock $\widetilde{P}^{m*}_t$ has only negligible impact on the variables, hence its graphical analysis is omitted.

Figure 6 displays the impacts of the cost-push shock $\tilde{\upsilon}_t$. The size of the shock is set at 0.08 per cent, hence it yields just 1 percentage point extra inflation at date $t = 1$ if the persistence is low. Beyond its positive impact on inflation, it negatively influences consumption and labor. The change of the inflation persistence parameters modifies the trajectories: the strong undershooting of the inflation rate is especially interesting in the high persistency case.

Figure 7 plots the impulse responses related to the nominal-wage shock $\tilde{\upsilon}^w_t$. The size of the shock is set in such a way that it induces 1 percentage point extra wage inflation at date $t = 1$ if the persistence is low. The reactions induced by $\tilde{\upsilon}^w_t$ are similar to that of $\tilde{\upsilon}_t$. However, it induces a greater increase of wage inflation, and a much smaller rise of inflation. Furthermore, the real wage level increases, and the decline of consumption, exports, and labor utilization are smaller.

The impulse responses generated by the productivity shock $\widetilde{A}_t$ can be found in *Figure 7*. A rise of productivity negatively influences inflation, this results in appreciation of the real exchange rate, since the nominal exchange rate is fixed. Thus, equation (17) implies that consumption rises. But this boom is relatively small compared to the size of productivity growth: hence the net effect of these two factors on labor demand is negative. Equation (26) implies that increasing productivity coincides with increasing $\tilde{c}_t^{wr}$, hence, although consumption increases, the consumption gap declines. As Galí (2002) discussed, it is a general feature of New Keynesian models that a rise of productivity results in decreasing labor demand. Moreover, this can be supported by recent empirical studies, although these findings sharply contradict the predictions of *real business cycle* (RBC) literature.

4.2 Analysis of the optimal solution

In this model there is only one policy variable: the monetary authority decides how to peg the nominal exchange rate at the accession date. Formally, this means that the depreciation rate $d\tilde{e}_1$ is a decision variable, but $d\tilde{e}_t = 0$, for all $t \geq 2$.

The optimal conversion rate is a result of the following optimization problem: policy makers have to choose $d\tilde{e}_1$ in order to maximize objective function (24), subject to the equations determining the trajectories of endogenous variables, and given Y_0, S_1, i.e. the vector of the state variables and the exogenous shocks, respectively.

It is shown in *Appendix B.3* that the solution of the above described problem yields the following expression:

$$d\tilde{e}_1 = -\frac{\sum_{j=1}^{6} \mathcal{K}_j^Y Y_0(j) + \sum_{s=2}^{9} \mathcal{K}_s^S S_1(s)}{\mathcal{K}_1^S},$$

the formulas for the coefficients in the above expression are also presented in the *Appendix*.

If, keeping everything else fixed, the jth state variable is above its steady-state value by 1 per cent, then decision makers should devaluate the nominal exchange rate by $-\mathcal{K}_j^Y/\mathcal{K}_1^S$ per cent in order to settle the optimal conversion rate. Similarly if, keeping everything else fixed, the sth shock variable is above its steady-state value by 1 per cent, then a $-\mathcal{K}_s^S/\mathcal{K}_1^S$ per cent devaluation is the optimal response.

Effects of the state variables

Table 2 displays the optimal-policy multipliers corresponding to the state variables $(-\mathcal{K}_j^Y/\mathcal{K}_1^S)$. The first row of the table contains the results of the model version with low inflation persistency, while the second row belongs to the version with high persistency. The multipliers of the inflation rate are expressed in annualized terms. A positive number in the table refers to a required devaluation.[10]

Table 2

Optimal multipliers of the state variables

Degree of inflation persistence	State variables				
	$\tilde{q}_0^d$	π_0	$\tilde{c}_0$	$\tilde{x}_0$	$\widetilde{w}_0$
low	-1	0.181	-0.032	-0.044	0.386
high	-1	0.706	-0.056	-0.086	0.534

Note: A positive entry refers to a required devaluation as an optimal policy response.

Following the discussion in the previous section, the value of the multiplier of $\tilde{q}_t^d$ comes as no surprise. It means that if the domestic component of the real exchange rate $\tilde{q}_t^d$, which represents practical misalignment indices in our model, is appreciated by 1 percent at date $t = 0$, then the decision makers' optimal response is a 1 per cent devaluation of the nominal exchange rate at date $t = 1$. This action would perfectly neutralize the effects of real exchange rate misalignment.

At first sight, it is surprising that the multiplier of the inflation rate is positive. This means that if the initial inflation rate is positive, then policy makers have to respond with a devaluation. The reason for this is that in the social welfare function (24) π_t and π_t^w do not appear themselves, rather their divergence from the past inflation rate. Thus, a too quick disinflation decreases welfare, just like a further increase of inflation. Hence, the optimal solution is moderate disinflation.

But, still it might be asked, how is it possible to reconcile any kind of disinflation with a depreciation? The answer is the following. If the inflation rate is given, then an exchange rate policy is neutral, if it devaluates at the same rate as that of the inflation. In this case the real exchange rate remains constant. If the inherited inflation rate is positive, then fixing the nominal exchange rate results in real depreciation, since the inflation rate does not become immediately zero. It has very similar effect if one fixes the nominal

[10] Variable $\hat{c}_t$ is absent from the table. Although formally it is a state variable, it does not affect any other variables, since it appears only in equation (27). Thus, the corresponding policy multiplier is equal to zero.

exchange rate after a mild devaluation, it yields real appreciation, a negative consumption gap and disinflation.[11]

According to *Table 2*, if consumption is above its steady-state value by 1 per cent, then the optimal reaction is a revaluation. After the inspection of *Figure 2* this becomes clear, since all three variables entering the social welfare function, i.e. the consumption gap, price and wage inflation arrive above their steady-state values.

The calculations reveal that if the initial real wage is above its steady-state value at date 0, then the decision makers should devaluate. *Figure 3* reveals the reason for this: the deviation of the real wage level yields a small increase of CPI inflation, a substantial decrease of wage inflation, and a negative consumption gap.

Numerical values of the optimal multipliers change significantly if one modifies the persistency parameters. It comes as no surprise that the multiplier of the inflation rate changes the most, in the high persistency version it becomes nearly four times bigger than in the low persistency case. But multipliers of other variables are nearly doubled as well.

Effects of the exogenous shocks

Let us study the optimal-policy multipliers corresponding to the exogenous shocks ($-\mathcal{K}_s^S/\mathcal{K}_1^S$). The results are summarized in *Table 3*. Shocks $\tilde{v}_t$ and $\tilde{v}_t^w$ are normalized in such a way that they induce 1 percent extra price or wage inflation in the baseline version of the model.

Table 3

Optimal multipliers of the exogenous shocks

Degree of inflation persistence	Exogenous shocks							
	$\tilde{c}_1^*$	$\tilde{c}_0^*$	$\tilde{x}_1^*$	$\widetilde{P}_1^{F*}$	$\widetilde{P}_1^{m*}$	$\tilde{v}_1$	$\tilde{v}_1^w$	$\widetilde{A}_1$
low	-0.796	-0.566	-1.242	-0.604	0.012	0.050	-0.750	0.560
high	-1.201	-0.747	-1.554	-0.799	0.137	0.581	-0.951	0.560

Note: A positive entry refers to a required devaluation as an optimal policy response.

Remarkably large multipliers belong to the contemporary and past realizations of the foreign business-cycle shock $\tilde{c}_t^*$ and the export-demand shock $\tilde{x}_t^*$. Positive shocks require revaluations as an optimal response in both cases,

[11] This argument, of course, does not mean that disinflation with nominal revaluation is necessarily a faulty policy. For example, it is possible that a given country can only meet the deadline of the Maastricht criteria of the Monetary Union if it performs such a radical disinflation that can be reconciled only with exchange rate revaluation.

since, as was discussed in *section 4.1*, they induce a rise in the consumption gap, as well as CPI and wage inflation: see *Figure 5*.

The foreign-CPI shock $\widetilde{P}_t^{F*}$ also features a significant multiplier, and the optimal reaction for a positive shock is also a revaluation. The reason is the same as previously. On the other hand, the effect of the import-price shock $\widetilde{P}_t^{m*}$ is negligible, and its multiplier has an opposite sign.

There are two types of cost-push shocks in the model: $\tilde{\upsilon}_t$ is the shock of the price setting process, $\tilde{\upsilon}_t^w$ is that of the wage setting process. If persistency is high both have remarkable effects. On the other hand, if persistency is low, then the multiplier of $\tilde{\upsilon}_t$ is relatively small. It is interesting that the multipliers of the two shocks have opposite signs. A positive price shock requires a devaluation, as in the case of the import-price shock. The reason is the same as in the case of a positive inflation rate at date 0, discussed previously. However, a positive wage setting shock requires a revaluation, since the induced rate of wage inflation is higher than that of CPI inflation: see *Figure 7*.

A positive productivity shock $\widetilde{A}_t$ requires significant devaluation, since it reduces the output gap, see *Figure 8*, as was discussed in the previous section.

Note that again most of the result are quite sensitive to the persistency parameter of the inflation process, the only exception is the productivity shock.

Summary

To summarize this subsection, the domestic component of the real exchange rate has a key but not exclusive role in determining the optimal conversion rate. Its role is prominent, since its optimal multiplier is stable, independent of the parameter values of the model, and the value of its multiplier is significantly higher in absolute value than that of other variables.

On the other hand, a decision based exclusively on the initial real exchange rate would be suboptimal, since an undervalued or overvalued real exchange rate always coincides with deviations of other variables, and some of these variables have significant multipliers. The past inflation rate and the past real wage level are the most important state variables, while the shocks of the foreign business cycle, foreign CPI, nominal wage level and productivity are the most important exogenous factors.

It is important to note that the optimal-policy multipliers of most variables change significantly if persistency parameters are modified. Thus, for the right decision in a certain economy it would be necessary to know thoroughly the empirical characteristics of price and wage formation processes.

4.3 Practical issues

This section briefly reviews how one should apply the theoretical results of this study in practice.

To start with, I clarify the interpretation of the steady state of the model. In this model, as is usual in the business cycle literature, the long-run paths of the variables are filtered out. The object of the analysis is the cyclical behavior of the variables around the long-run trajectories, and not the long-run behavior. Hence, the steady state of the model represents these long-run paths, which are not modelled explicitly.

It is assumed that the long-run growth of the real variables in the model are determined by two factors: long-run technological progress and the convergence of per capita real income to the growth path of developed countries, i.e. the transitional dynamics. These processes can be properly captured by neoclassical open economy models of growth – see, e.g., Barro et al. (1992) – and long-run evolution of nominal variables are determined by long-run money supply.

It is demonstrated that one of the key determinants of the optimal conversion rate is a certain misalignment index, namely, the deviation of the domestic component of the real exchange rate from its steady state value, or in other words, its deviation from its long-run path. That is why it is important to identify empirically the long-run trajectory of this variable. The literature dealing with practical equilibrium real exchange rate estimations gives some guidelines. Although the notions of that literature are not perfectly compatible with the categories of general equilibrium neoclassical models of growth, one can try to reconcile them.

In their survey of equilibrium real exchange rate concepts Driver and Westaway (2003) define the long-run equilibrium as the point when net wealth is in full stock-flow equilibrium, so that changes to asset stocks are zero. In a neoclassical model of growth this happens when the transition process is over, i.e. per capita real incomes are equalized. Since the steady state of my model does not represent this state of the economy, it rather corresponds to the medium-run equilibrium concepts of practical misalignment calculations.

The key nominal endogenous variable of this model is the inflation rate. In the model its steady state value is zero. But in reality the long-run inflation rate is usually positive, in the EU it is around two percent. That is, it is useful to add two percentage points to the inflation numbers of this paper.

One may criticize my result related to the inflation rate as irrelevant, since the Maastricht criteria require that the inflation rate of accession countries cannot be significantly higher than their long-run values. But this study demonstrates that if inflation persistency is high, then at the end of a disinflation process there can occur serious undershooting of the long-run value of inflation. Due to procedural reasons, at the accession date it is improbable that the conversion rate can be modified as much as optimality would require. Thus, if in a given economy the persistence of inflation is high or uncertain, then it is better to start the disinflation process long before the accession. If there is enough time for disinflation, then the potential undershooting will disappear prior to the accession date, and the inflation rate will be close enough to its long-run value. Hence, a nominal-exchange-rate alignment would not be necessary when the country joins the Monetary Union.

Some limitations of the model for direct policy application have to be mentioned.

First, the conversion rates of accession countries are a result of a multilateral decision, and the welfare of all the concerned countries is taken into account: but in this model the welfare of only one accession country is considered.

Second, it is not taken into account in this model that accession countries can be viewed as competing peripheral countries, which trade with the center of the EU. In this case, as it was shown in the paper of Corsetti et al. (1999), a given nominal exchange rate movement can have different effects depending on the relative weight of trade with the center and with other peripheral countries.

Third, in the model there is no unemployment. However, Világi (2004b) demonstrates that some welfare implications of NOEM models are sensitive to the assumptions about the labor market.

Fourth, in this model the government's budget is always balanced. However, a budget deficit may have inflationary effects, see, e.g., Woodford (2001). Thus, it has implications for the determination of the conversion rate.

Finally, the Balassa – Samuelson (BS) effect (i.e. the productivity induced divergence of sectoral inflation rates and the accompanying real appreciation) is not considered here. Világi (2004a) reviewed its empirical significance, and it may have implications for the choice of the optimal conversion rate: as Aoki (2001) and Benigno (2001) demonstrated, the optimal policy should put more weight to stabilization of the inflation rate in the sector with stronger nominal rigidities. Thus, in economies with diverging sectoral inflation rates, and with significantly different sectoral rigidities it is not sufficient to consider only the average inflation rate for the determination of the optimal conversion rate.

The reason why the BS effect is neglected is that in order to derive a tractable social welfare function one has to impose restrictions on the co-movement of consumption and export. (See the details in *section 3.1* and

Appendix B.2.) The restrictions applied in this model contradict the pricing to market assumption, which is necessary in a NOEM model to generate the BS effect, as it was demonstrated in Világi (2004a).

5 Conclusions

This paper has examinded how a country joining the European Economic and Monetary Union should choose its conversion rate. It was shown that, contrary to the widespread approach of the non-academic economic policy literature, it is not enough to base this decision exclusively on one factor, namely, the real exchange rate. It was demonstrated that although the misalignment of the real exchange rate was a key factor in the determination of the conversion rate, it did not have an exclusive role in determining the optimal conversion rate.

A proper misalignment index is proved to be a robust, parameter independent, and significant factor. On the other hand, the inflation rate and the real wage level are another key state variables which have to be taken into consideration for the determination of the conversion rate. Furthermore, the foreign-business-cycle, foreign-price, nominal-wage and productivity shocks are exogenous factors also containing significant information for proper policy decision-making.

The importance of using a model-based social welfare function instead of ad-hoc welfare criteria was also demonstrated. Due to the persistence of inflation process the exact social welfare function derived from the model contains both contemporary and past rates of inflation. As a consequence, the optimal policy reaction to some variables substantially differs from that derived from models with ad-hoc policy objective functions.

Furthermore, it was shown that the optimal exchange rate policy changed significantly if persistency of CPI and wage inflation were modified. Thus, for the right decision on the conversion rate in a certain economy it is necessary to have a thorough knowledge of the empirical characteristics of price and wage setting processes.

A Appendix

A.1 The steady state

In this section the non-stochastic steady state of the model is described. Variables without time indices refer to their steady-state values.

The labor and imports demand functions, i.e. equations (12) have the following form in the steady state:

$$l^{1-\alpha} = a\frac{c}{A}, \quad m^{1-\alpha} = (1-a)\frac{c}{A},$$

where it is used that in the steady state all firms have the same level of production and input demand. It is assumed that $x/A = m^{1-\alpha}$, as a consequence, $x = c(1-a)/a$, or $l^{1-\alpha} = c/A$. Thus, the labor demand is the same as in a closed economy with similar technology, but $z = l$.

Furthermore, it is assumed that in the steady state international trade is balanced, hence $Px = eP^{m*}m$. Let us take as given the share of exports in GDP:

$$s^x = \frac{Px}{Pc + Px - eP^{m*}m} = \frac{x}{c} = \frac{1-a}{a}.$$

According to Hungarian data approximately $s^x = 0.6$, hence $a = 0.625$ and $x = 0.6c$. Coefficients a_l and a_m can be calculated as $a = a_l^{1-\alpha}$ and $(1-a) = a_m^{1-\alpha}$. Thus, $a_l = 0.593$ and $a_m = 0.336$.

It is assumed that in the steady state c is equal to the welfare maximizing consumption level of the above closed economy. The social welfare maximizing allocation can be given by the solution of the following optimization problem:

$$\max_{c_t} \sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_1 \left[u(c_t - hc_{t-1}) - v\left(c_t^{\bar{\alpha}} A_t^{-\bar{\alpha}}\right) \right],$$

where $\bar{\alpha} = (1-\alpha)^{-1}$. The corresponding first-order condition in the steady-state is

$$u'(c(1-h))(1-\beta h) = \bar{\alpha} v'\left(c^{\bar{\alpha}} A^{-\bar{\alpha}}\right)) c^{\hat{\alpha}} A^{\bar{\alpha}}, \tag{28}$$

where $\hat{\alpha} = \alpha\bar{\alpha}$. This implies that

$$1 = \bar{\alpha}\frac{(1-h)^{\sigma}}{1-\beta h} c^{\sigma+\varphi\bar{\alpha}+\hat{\alpha}} A^{-(1+\varphi)\bar{\alpha}}.$$

For the sake of simplicity, let us choose A such that $1 = (c+x)^{\hat{\alpha}} A^{-\bar{\alpha}}$, i.e. $(c+x)^{\alpha} = A$. $x = (1-a)c/a$, and this implies that $A = (c/a)^{\alpha}$. Substituting this into the previous expression yields

$$1 = \bar{\alpha}\frac{(1-h)^{\sigma}}{1-\beta h} c^{\sigma+\varphi} a^{(1+\varphi)\hat{\alpha}}.$$

Thus, $c = 1.224$ and $A = 1.070$. Using $l^{1-\alpha} = c/A$ one obtains $l = 1.618$.

The steady-state form of the labor supply equation is

$$\frac{W}{P} = \mu^w \left((1-h)c\right)^\sigma l^\varphi,$$

where it is assumed that $\mu^w = \theta_w/(\theta_w - 1) = 1.5$ and $P = 1$. Hence, $W = 0.665$.

Balanced international trade implies that

$$\frac{P^{m*}}{P} = Ax^{-\hat{\alpha}},$$

where I used that $e = 1$ and $m = x^{\bar{\alpha}}/A$. Knowing c one can calculate that $x = 0,735$, hence $P^{m*} = 1,107$.

The steady-state form of the price setting equation is

$$1 = \mu\bar{\alpha}(c+x)^{\hat{\alpha}} A^{-\bar{\alpha}} \left(a_l \frac{W}{P} + a_m \frac{P^{m*}}{P}\right).$$

There is only one value of μ which satisfies this equation, since $\bar{\alpha}$ is a given parameter, and it is assumed that $(c+x)^{\hat{\alpha}} A^{-\bar{\alpha}} = 1$, and a_l, a_m, W, P^{m*}/P were calculated previously. Let us assume that the government sets the tax/transfer variable τ in such a way that the price setting equation is satisfied, hence $\mu = 1.174$.

A.2 Second-order approximation of the social welfare function

Following Woodford (2003, ch. 6), this section provides a second-order Taylor approximation of the social welfare function, which is the aggregate utility function of households. The social welfare function is the following:

$$\mathcal{U}(Y_0, S) = \sum_{t=1}^{\infty} \beta^{t-1} \left[u(H_t) - \int_0^1 v(l_t(j))\,dj\right], \tag{29}$$

where Y_0 is the vector of the date 0 state variables, S=$[S_1, S_2, S_3, \ldots]$, and S_t is the vector of the shock variables at date t. The Taylor approximation of the consumption term around the steady state is

$$\begin{aligned} u(H_t) &= u'(H)\left(\Delta c_t - h\Delta c_{t-1}\right) + \frac{1}{2}u''(H)\left[(\Delta c_t)^2 + h^2 (\Delta c_{t-1})^2\right] \\ &- hu''(H)\Delta c_t \Delta c_{t-1} + \text{t.i.p.} + o\left(||S||^3\right), \end{aligned}$$

where $\Delta c_t = c_t - c$, $o\left(||S||^3\right)$ contains the third and higher order error terms, and "t.i.p." means *terms that are independent of policy*, which are constant terms and exogenous variables. Obviously, exchange rate policy does not affect these terms.

Expressions

$$\begin{aligned} \Delta c_t &= c\left(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2\right) + o\left(||S||^3\right), \\ (\Delta c_t)^2 &= c\tilde{c}_t^2 + o\left(||S||^3\right) \end{aligned}$$

imply that

$$\begin{aligned} u(H_t) &= u'(H)c\left(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2\right) - hu'(H)c\left(\tilde{c}_{t-1} + \frac{1}{2}\tilde{c}_{t-1}^2\right) \\ &+ u''(H)c^2\left(\frac{1}{2}\tilde{c}_t^2 + \frac{1}{2}h^2\tilde{c}_{t-1}^2 - h\tilde{c}_t\tilde{c}_{t-1}\right) + \text{t.i.p.} + o\left(||S||^3\right), \end{aligned}$$

where the tilde denotes the log-deviation of a variable from its steady-state value. Rearranging the above expression yields

$$\begin{aligned} u(H_t) &= u'(H)c\left(\tilde{c}_t - h\tilde{c}_{t-1}\right) + \frac{1}{2}u'(H)c\left(\tilde{c}_t^2 - h\tilde{c}_t^2\right) \\ &+ \frac{1}{2}u''(H)c^2\left(\tilde{c}_t - \tilde{c}_{t-1}\right)^2 + \text{t.i.p.} + o\left(||S||^3\right). \end{aligned}$$

Using the fact that

$$\frac{u''(H)c}{u'(H)} = \frac{-\sigma(1-h)^{-\sigma-1}c^{-\sigma}}{(1-h)^{-\sigma}c^{-\sigma}} = \frac{-\sigma}{1-h}$$

one can obtain

$$\begin{aligned} u(H_t) &= u'(H)c\left\{\tilde{c}_t - h\tilde{c}_{t-1} + \frac{1}{2}\left[\tilde{c}_t^2 - h\tilde{c}_{t-1}^2 - \frac{\sigma}{1-h}\left(\tilde{c}_t - h\tilde{c}_{t-1}\right)^2\right]\right\} \\ &+ \text{t.i.p.} + o\left(||S||^3\right). \end{aligned}$$

The discounted sum of the above expression is

$$\begin{aligned} \sum_{t=1}^{\infty}\beta^{t-1}u(H_t) &= (1-\beta h)u'(H)c\sum_{t=1}^{\infty}\beta^{t-1}\left\{\tilde{c}_t + \frac{1}{2}\left[1 - \left(1+\beta h^2\right)\bar{\sigma}\right]\tilde{c}_t^2\right\} \\ &+ (1-\beta h)u'(H)c\sum_{t=1}^{\infty}\beta^{t-1}h\tilde{c}_t\tilde{c}_{t-1} \\ &+ \text{t.i.p.} + o\left(||S||^3\right), \end{aligned} \tag{30}$$

where

$$\bar{\sigma} = \frac{\sigma}{(1-h)(1-\beta h)}.$$

Now let us consider the labor term of the social welfare function. If $l_t(i)$ is the labor input of firm i, then aggregate labor utilization is

$$l_t = \int_0^1 l_t(i)\, di = \mathrm{E}_i\left[l_t(i)\right].$$

This implies that

$$\tilde{l}_t = \mathrm{E}_i\left[\tilde{l}_t(i)\right] + \frac{1}{2}\mathrm{var}_i\left[\tilde{l}_t(i)\right] + o\left(||S||^3\right).$$

Recall equation (12),

$$l_t^{1-\alpha}(i) = n_t(i) = a\frac{y_t(i)}{A_t}.$$

Thus, $\tilde{l}_t(i) = \bar{\alpha}\tilde{n}_t(i)$, where $\bar{\alpha} = (1-\alpha)^{-1}$. As a consequence,

$$\tilde{l}_t = \bar{\alpha}\mathrm{E}_i\left[\tilde{n}_t(i)\right] + \frac{1}{2}\bar{\alpha}^2\mathrm{var}_i\left[\tilde{n}_t(i)\right] + o\left(||S||^3\right).$$

As for y_t, it is true for n_t that

$$n_t = \left(\int_0^1 n_t(i)^{\frac{\theta-1}{\theta}}\, di\right)^{\frac{\theta}{\theta-1}},$$

which implies that

$$\mathrm{E}_i\left[\tilde{n}_t(i)\right] = \tilde{n}_t - \frac{1}{2}\frac{\theta-1}{\theta}\mathrm{var}_i\left[\tilde{n}_t(i)\right] + o\left(||S||^3\right),$$

thus,

$$\tilde{l}_t = \bar{\alpha}\tilde{n}_t + \frac{1}{2}\bar{\alpha}\theta\left(1+\theta\hat{\alpha}\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right] + o\left(||S||^3\right), \tag{31}$$

where $\hat{\alpha} = \alpha\bar{\alpha}$, and it is used that $\mathrm{var}_i\left[\tilde{n}_t(i)\right] = \mathrm{var}_i\left[\tilde{y}_t(i)\right]$, and $\mathrm{var}_i\left[\tilde{y}_t(i)\right] = \theta^2\mathrm{var}_i\left[\widetilde{P}_t(i)\right]$, which is a consequence of equation (11).

Since

$$n_t = a\frac{y_t}{A_t} = a\frac{c_t + x_t}{A_t} = \frac{c_t}{A_t}a\left(1+\frac{x_t}{c_t}\right),$$

variable $\tilde{n}_t$ can be expressed as

$$\tilde{n}_t = \tilde{c}_t - \widetilde{A}_t - \tilde{g}_t,$$

where

$$\tilde{g}_t = \frac{x}{x+c}\left(\tilde{c}_t - \tilde{x}_t\right) = (1-a)\left(\tilde{c}_t - \tilde{x}_t\right).$$

The inspection of (17) and (18) reveals that condition

$$\eta = \frac{(1-h)(1-\beta h)}{\sigma} \tag{32}$$

ensures that the difference between $\tilde{c}_t$ and $\tilde{x}_t$ depends only on stochastic shocks and initial conditions. Furthermore, formula (12), a consequence of the Leontieff technology, and condition (32) imply that variable $\tilde{r}_t = \widetilde{A}_t + \tilde{g}_t$ also depends only on stochastic shocks and initial conditions. Substituting $\tilde{n}_t = \tilde{c}_t - \tilde{r}_t$ into equation (31) yields

$$\tilde{l}_t = \bar{\alpha}\left(\tilde{c}_t - \tilde{r}_t\right) + \frac{1}{2}\bar{\alpha}\theta\left(1+\theta\hat{\alpha}\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right] + o\left(||S||^3\right). \tag{33}$$

The approximation of the disutility of labor of household j is given by

$$v(l_t(j)) = v'(l)l\left\{\tilde{l}_t(j) + \frac{1}{2}(1+\varphi)\tilde{l}_t^2(j)\right\} + \text{t.i.p.} + o\left(||S||^3\right).$$

Aggregating this formula one obtains

$$\begin{aligned}&\int_0^1 v(l_t(j))\,dj = \\ &v'(l)l\left\{\mathrm{E}_j\left[\tilde{l}_t(j)\right] + \frac{1}{2}(1+\varphi)\left[\mathrm{E}_j\left[\tilde{l}_t(j)\right]^2 + \mathrm{var}_j\left[\tilde{l}_t(j)\right]\right]\right\} + \text{t.i.p.} + o\left(||S||^3\right).\end{aligned}$$

Using equations

$$\begin{aligned}\mathrm{E}_j\left[\tilde{l}_t(j)\right] &= \tilde{l}_t - \frac{1}{2}\frac{\theta_w - 1}{\theta_w}\mathrm{var}_j\left[\tilde{l}_t(j)\right] + o\left(||S||^3\right), \\ \mathrm{E}_j\left[\tilde{l}_t(j)\right]^2 &= \tilde{l}_t^2 + o\left(||S||^3\right),\end{aligned}$$

and $\mathrm{var}_j\left[\tilde{l}_t(j)\right] = \theta_w^2\mathrm{var}_j\left[\widetilde{w}_t(j)\right]$, an implication of equation (7), yields

$$\begin{aligned}&\int_0^1 v(l_t(j))\,dj = \qquad (34)\\ &v'(l)l\left\{\tilde{l}_t + \frac{1}{2}(1+\varphi)\tilde{l}_t^2 + \frac{1}{2}\theta_w(1+\varphi\theta_w)\mathrm{var}_j\left[\widetilde{W}_t(j)\right]\right\} + \text{t.i.p.} + o\left(||S||^3\right).\end{aligned}$$

Substitute equation (33) into equation (34),

$$\int_0^1 v(l_t(j))\,dj = \tag{35}$$
$$\frac{(1-\beta h)u'(H)c}{2}\left\{2\tilde{c}_t + (1+\varphi)\bar{\alpha}\left(\tilde{c}_t^2 - 2\tilde{r}_t\tilde{c}_t\right) + \theta\left(1+\hat{\alpha}\theta\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right]\right\}$$
$$+\frac{(1-\beta h)u'(H)c}{2}\left\{\theta_w\bar{\alpha}^{-1}\left(1+\varphi\theta_w\right)\mathrm{var}_j\left[\widetilde{W}_t(j)\right]\right\} + \text{t.i.p.} + o\left(||S||^3\right),$$

where it is used that

$$(1-\beta h)u'(H)c = \bar{\alpha}v'(l)l,$$

since equation (28) implies that

$$(1-\beta h)u'(H)c = \bar{\alpha}v'(l)c^{\hat{\alpha}+1}A^{-\bar{\alpha}} = \bar{\alpha}v'(l)\left(\frac{c}{A}\right)^{\bar{\alpha}}.$$

Combining equations (30) and (35) yields the following expression for the utility function defined by equation (29):

$$\begin{aligned}\mathcal{U}(Y_0,S) = & - J\sum_{t=1}^{\infty}\beta^{t-1}\left\{\left[\left(1+\beta h^2\right)\bar{\sigma} + \varphi\bar{\alpha} + \hat{\alpha}\right]\tilde{c}_t^2 - 2h\bar{\sigma}\tilde{c}_t\tilde{c}_{t-1}\right\} \\ & - J\sum_{t=1}^{\infty}\beta^{t-1}\left\{2(1+\varphi)\bar{\alpha}\tilde{r}_t\tilde{c}_t + \theta\left(1+\theta\hat{\alpha}\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right]\right\} \\ & - J\sum_{t=1}^{\infty}\beta^{t-1}\theta_w\bar{\alpha}^{-1}\left(1+\theta_w\varphi\right)\mathrm{var}_j\left[\widetilde{W}_t(j)\right] + \text{t.i.p.} + o\left(||S||^3\right),\end{aligned}$$

where $J = (1-\beta h)u'(H)c/2$. For the calculation of the coefficient of term $\tilde{c}_t^2$ it was taken into account that $(1+\varphi)\bar{\alpha} = 1 + \varphi\bar{\alpha} + \hat{\alpha}$.

As Woodford (2003, ch. 6) shows, the welfare analysis becomes inaccurate, if the approximation of the social welfare function contains first-order terms of endogenous variables. Condition (28), formula (12), an implication of the Leontieff technology, and condition (32) ensure that in the objective function there are only second-order terms of endogenous variables.

Define $\tilde{c}_t^{wr}$, the *welfare reference* level of consumption, which is determined by

$$\left[\left(1+\beta h^2\right)\bar{\sigma} + \varphi\bar{\alpha} + \hat{\alpha}\right]\tilde{c}_t^{wr} - \beta h\bar{\sigma}\mathrm{E}_t\left[\tilde{c}_{t+1}^{wr}\right] - h\bar{\sigma}\tilde{c}_{t-1}^{wr} = (1+\varphi)\bar{\alpha}\tilde{r}_t.$$

Using this definition one can express $\mathcal{U}(Y_0, S)$ as

$$\begin{aligned}\mathcal{U}(Y_0,S) = & - J\sum_{t=1}^{\infty}\beta^{t-1}\left[\left(1+\beta h^2\right)\bar{\sigma}+\varphi\bar{\alpha}+\hat{\alpha}\right]\left\{\tilde{c}_t^2-2\tilde{c}_t^{wr}\tilde{c}_t\right\} \\ & - J\sum_{t=1}^{\infty}\beta^{t-1}2h\bar{\sigma}\left\{\tilde{c}_t\tilde{c}_{t-1}-\tilde{c}_t\left(\beta\tilde{c}_{t+1}^{wr}+\tilde{c}_{t-1}^{wr}\right)\right\} \\ & - J\sum_{t=1}^{\infty}\beta^{t-1}\theta\left(1+\theta\hat{\alpha}\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right] \\ & - J\sum_{t=1}^{\infty}\beta^{t-1}\theta_w\bar{\alpha}^{-1}\left(1+\theta_w\varphi\right)\mathrm{var}_j\left[\widetilde{W}_t(j)\right]+\text{t.i.p.}+o\left(||S||^3\right).\end{aligned} \tag{36}$$

The expression

$$-J\sum_{t=1}^{\infty}\beta^{t-1}\left\{\left[\left(1+\beta h^2\right)\bar{\sigma}+\varphi\bar{\alpha}+\hat{\alpha}\right]\left(\tilde{c}_t^{wr}\right)^2+\tilde{c}_t^{wr}\tilde{c}_{t-1}^{wr}\right\}+2h\bar{\sigma}\tilde{c}_0\tilde{c}_1^{wr}, \tag{37}$$

depends only on the exogenous variable $\tilde{r}_t$ and initial conditions, that is, only on terms that are independent of policy (t.i.p.). Let us define the *consumption gap*,

$$\hat{c}_t = \tilde{c}_t - \tilde{c}_t^{wr}.$$

Using this definition, and adding the discounted sum (37) to equation (36) yields

$$\begin{aligned}\mathcal{U}(Y_0,S) = & - J\sum_{t=1}^{\infty}\beta^{t-1}\left\{\mathsf{A}\hat{c}_t^2-2\mathsf{B}\hat{c}_t\hat{c}_{t-1}\right\} \\ & - J\sum_{t=1}^{\infty}\beta^{t-1}\theta\left(1+\theta\hat{\alpha}\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right] \\ & - J\sum_{t=1}^{\infty}\beta^{t-1}\theta_w\bar{\alpha}^{-1}\left(1+\theta_w\varphi\right)\mathrm{var}_j\left[\widetilde{W}_t(j)\right]+\text{t.i.p.}+o\left(||S||^3\right),\end{aligned} \tag{38}$$

where $\mathsf{A} = (1+\beta h^2)\,\bar{\sigma}+\varphi\bar{\alpha}+\hat{\alpha}$ and $\mathsf{B} = h\bar{\sigma}$. Define δ_0 and δ in such a way that

$$\mathsf{A} = (1+\beta\delta^2)\delta_0, \quad \text{and} \quad \mathsf{B} = \delta\delta_0.$$

Then parameter δ is the solution of the following quadratic equation:

$$\mathsf{A}\delta = \mathsf{B}\left(1+\beta\delta^2\right).$$

Let us choose the smaller root of the above equation, which satisfies $0 \leq \delta \leq h$. Equation (38) can be simplified further, since

$$\begin{aligned}
\sum_{t=1}^{\infty} \beta^{t-1} \left(\mathsf{A}\hat{c}_t^2 - 2\mathsf{B}\hat{c}_t\hat{c}_{t-1}\right) &= \delta_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\left(1+\beta\delta^2\right)\hat{c}_t^2 - 2\delta\hat{c}_t\hat{c}_{t-1}\right] \\
&= \delta_0 \sum_{t=1}^{\infty} \beta^{t-1} \left(\hat{c}_t^2 - 2\delta\hat{c}_t\hat{c}_{t-1} + \delta^2\hat{c}_{t-1}^2\right) + \delta_0\delta^2\hat{c}_0 \\
&= \delta_0 \sum_{t=1}^{\infty} \beta^{t-1} \left(\hat{c}_t - \delta\hat{c}_{t-1}\right)^2 + \text{t.i.p.}
\end{aligned}$$

Substituting the above expression into (38) yields

$$\begin{aligned}
\mathcal{U}(Y_0, S) = & - J\sum_{t=1}^{\infty} \beta^{t-1}\delta_0 \left(\hat{c}_t - \delta\hat{c}_{t-1}\right) \qquad (39) \\
& - J\sum_{t=1}^{\infty} \beta^{t-1}\theta\left(1+\theta\hat{\alpha}\right)\operatorname{var}_i\left[\widetilde{P}_t(i)\right] \\
& - J\sum_{t=1}^{\infty} \beta^{t-1}\theta_w\bar{\alpha}^{-1}\left(1+\theta_w\varphi\right)\operatorname{var}_j\left[\widetilde{W}_t(j)\right] + \text{t.i.p.} + o\left(||S||^3\right).
\end{aligned}$$

Now it will be shown how it is possible to express the variance of prices and wages by price and wage inflation. First, the assumptions on pricing behavior imply that

$$\begin{aligned}
\mathrm{E}_i\left[\widetilde{P}_t(i)\right] &= \gamma\mathrm{E}_i\left[\widetilde{P}_{t-1}(i) + \vartheta\pi_{t-1}\right] + (1-\gamma)\widetilde{P}_t^o, & (40) \\
\mathrm{E}_i\left[\widetilde{P}_t^2(i)\right] &= \gamma\mathrm{E}_i\left[\left(\widetilde{P}_{t-1}(i) + \vartheta\pi_{t-1}\right)^2\right] + (1-\gamma)\left(\widetilde{P}_t^o\right)^2, & (41)
\end{aligned}$$

where $\widetilde{P}_t^o$ denotes the price, which is chosen by those firms which set their price in an optimal, forward-looking way in period t.

Let $\bar{P}_t = \mathrm{E}_i\left[\widetilde{P}_t(i)\right]$, then by equation (40) it is easy to show that

$$\bar{P}_t - \bar{P}_{t-1} - \vartheta\pi_{t-1} = (1-\gamma)\left(\widetilde{P}_t^o - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right). \qquad (42)$$

Now let us express the variance of prices,

$$
\begin{aligned}
\mathrm{var}_i\left[\widetilde{P}_t(i)\right] &= \mathrm{var}_i\left[\widetilde{P}_t(i)-\bar{P}_{t-1}-\vartheta\pi_{t-1}\right] \\
&= \mathrm{E}_i\left[\left(\widetilde{P}_t(i)-\bar{P}_{t-1}-\vartheta\pi_{t-1}\right)^2\right]-\mathrm{E}_i\left[\widetilde{P}_t(i)-\bar{P}_{t-1}-\vartheta\pi_{t-1}\right]^2 \\
&= \mathrm{E}_i\left[\left(\widetilde{P}_t(i)-\bar{P}_{t-1}-\vartheta\pi_{t-1}\right)^2\right]-\left(\bar{P}_t-\bar{P}_{t-1}-\vartheta\pi_{t-1}\right)^2 \\
&= \gamma\mathrm{E}_i\left[\left(\widetilde{P}_{t-1}(i)-\bar{P}_{t-1}\right)^2\right]+(1-\gamma)\left(\widetilde{P}_t^o-\widetilde{P}_{t-1}-\vartheta\pi_{t-1}\right)^2 \\
&- \left(\bar{P}_t-\bar{P}_{t-1}-\vartheta\pi_{t-1}\right)^2,
\end{aligned}
$$

where equations (40) and (41) are used for the derivation of the last equality. Using the above formula and equation (42) yields

$$\mathrm{var}_i\left[\widetilde{P}_t(i)\right]=\gamma\mathrm{var}_i\left[\widetilde{P}_{t-1}(i)\right]+\frac{\gamma}{1-\gamma}(\bar{P}_t-\bar{P}_{t-1}-\vartheta\pi_{t-1})^2.$$

Since $\bar{P}_t=\widetilde{P}_t+o\left(||S||^2\right)$, one can obtain

$$\left(\bar{P}_t-\bar{P}_{t-1}-\vartheta\pi_{t-1}\right)^2=\left(\pi_t-\vartheta\pi_{t-1}\right)^2+o\left(||S||^3\right).$$

Combining the previous formulas yields the following expression for the variance of prices:

$$\mathrm{var}_i\left[\widetilde{P}_t(i)\right]=\gamma\mathrm{var}_i\left[\widetilde{P}_{t-1}(i)\right]+\frac{\gamma}{1-\gamma}(\pi_t-\vartheta\pi_{t-1})+o\left(||S||^3\right). \tag{43}$$

One can show that a similar expression is true for the variance of wages, i.e.

$$\mathrm{var}_j\left[\widetilde{W}_t(j)\right]=\gamma_w\mathrm{var}_j\left[\widetilde{W}_{t-1}(j)\right]+\frac{\gamma_w}{1-\gamma_w}(\pi_t^w-\vartheta_w\pi_{t-1})+o\left(||S||^3\right). \tag{44}$$

Using equation (43) by recursive substitutions it is possible to show that

$$\sum_{t=1}^{\infty}\beta^{t-1}\mathrm{var}_i\left[\widetilde{P}_t(i)\right]=\frac{\gamma\sum_{t=1}^{\infty}\beta^{t-1}}{(1-\gamma)(1-\beta\gamma)}(\pi_t-\vartheta\pi_{t-1})^2+\text{t.i.p.}+o\left(||S||^3\right). \tag{45}$$

Similarly equation (44) implies that

$$
\begin{aligned}
&\sum_{t=1}^{\infty}\beta^{t-1}\mathrm{var}_j\left[\widetilde{W}_t(j)\right]= \qquad (46)\\
&\frac{\gamma_w\sum_{t=1}^{\infty}\beta^{t-1}}{(1-\gamma_w)(1-\beta\gamma_w)}(\pi_t^w-\vartheta_w\pi_{t-1})^2+\text{t.i.p.}+o\left(||S||^3\right).
\end{aligned}
$$

Substitute equations (45) and (46) into equation (39), and use definitions (14) and (9), then

$$
\begin{aligned}
\mathcal{U}(Y_0, S) &= -J \sum_{t=1}^{\infty} \beta^{t-1} \delta_0 \hat{c}_t^2 \\
&-J \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \frac{\theta}{\xi}(\pi_t - \vartheta \pi_{t-1}) + \frac{\bar{\alpha}^{-1}\theta_w}{\xi_w}(\pi_t^w - \vartheta_w \pi_{t-1}) \right\} + \text{t.i.p.} + o\left(||S||^3\right).
\end{aligned}
$$

Let $\mathcal{J} = J(\theta/\xi + (1-\alpha)\theta_w/\xi_w)$, then the social welfare function can be expressed as

$$
\mathcal{U}(Y_0, S) = -\mathcal{J} \sum_{t=1}^{\infty} \beta^{t-1} L_t + o\left(||S||^3\right) + \text{t.i.p.},
$$

where

$$
L_t = \lambda_c \hat{c}_t^2 + \lambda_\pi (\pi_t - \vartheta \pi_{t-1})^2 + \lambda_w (\pi_t^w - \vartheta_w \pi_{t-1})^2,
$$

and

$$
\begin{aligned}
\lambda_c &= \frac{\delta_0}{\theta \xi^{-1} + (1-\alpha)\theta_w \xi_w^{-1}}, \quad \lambda_\pi = \frac{\theta \xi^{-1}}{\theta \xi^{-1} + (1-\alpha)\theta_w \xi_w^{-1}}, \\
\lambda_w &= \frac{(1-\alpha)\theta_w \xi_w^{-1}}{\theta \xi^{-1} + (1-\alpha)\theta_w \xi_w^{-1}}.
\end{aligned}
$$

This obviously implies that maximization of the expected utility function

$$
\mathrm{E}_1 \left[\mathcal{U}(Y_0, S)\right]
$$

is equivalent to the minimization of the expected loss function

$$
\sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_1 \left[L_t\right].
$$

A.3 Calculation of the optimal solution

In this model there is only one policy variable: the monetary authority determines the value of the nominal exchange rate at date $t = 1$, and it remains unchanged later. Formally, this means that the depreciation rate $d\tilde{e}_1$ is a decision variable, but $d\tilde{e}_t = 0$, for all $t \geq 2$. Joining a currency union is not simply an exchange rate peg, but it means that pegging the exchange rate is perfectly credible, hence rational expectations imply that $\mathrm{E}_1 \left[d\tilde{e}_t\right] = 0$, for all $t \geq 2$.

This implies that it is worthwhile treating the depreciation rate formally as a first-order autoregressive process, with autoregressive parameter $\phi_e = 0$, and the realizations of its innovations $\epsilon_t^e = 0$, for all $t \geq 2$. That is,

$$d\tilde{e}_t = \phi_e^{t-1} d\tilde{e}_{t-1} + \epsilon_t^e, \quad \mathrm{E}_1 \left[d\tilde{e}_t \right] = \phi_e^{t-1} d\tilde{e}_1.$$

Thus the exogenous variables, namely, the policy variable and the stochastic shocks, are treated uniformly in the model.

All the stochastic shocks of the model are determined by first-order autoregressive processes, and it is assumed that the shocks are uncorrelated to each other. To simplify the calculations in equation (17) $\tilde{c}_{t-1}$ is replaced by

$$\tilde{c}_{t-1} = \frac{\tilde{c}_t - \epsilon_t^{c*}}{\phi_{c*}},$$

where ϕ_{c*} and ϵ_t^{c*} are the corresponding autoregressive parameter and innovation, respectively. The vector of the shocks is

$$S_t = [d\tilde{e}_t,\, \tilde{c}_t^*,\, \epsilon_t^{c*},\, \tilde{x}_t^*,\, \widetilde{P}_t^{F*},\, \widetilde{P}_t^{m*},\, \widetilde{A}_t,\, \tilde{\upsilon}_t,\, \tilde{\upsilon}_t^w]'.$$

The evolution of the exogenous variables is described by process

$$S_t = \Phi S_{t-1} + \mathcal{E}_t, \tag{47}$$

where coefficient matrix Φ is diagonal.

Let us supplement the log-linearized model of equations (17) – (23) with equation (27). This system of equations is solved by the undetermined coefficients algorithm. The output of the algorithm is the set of Q, $\bar{Q}$, Ω, and $\bar{\Omega}$ matrices, which are used to determine the paths of the endogenous variables,

$$Y_t = QY_{t-1} + \Omega S_t, \qquad \bar{Y}_t = \bar{Q} Y_{t-1} + \bar{\Omega} S_t, \tag{48}$$

where

$$Y_t = \left[\tilde{c}_t,\, \hat{c}_t\ \pi_t,\, \widetilde{w}_t,\, \tilde{x}_t,\, \tilde{q}_t^d \right]'$$

is the vector of the state variables, $\bar{Y}_t$ is the vector of other endogenous variables. It is required that the eigenvalues of matrix Q are smaller than 1 in absolute value. Using equations (47) and (48) one can show by recursive substitutions that

$$\mathrm{E}_1 \left[Y_t \right] = K_t Y_0 + G_t S_1, \tag{49}$$

where

$$K_t = Q^t, \quad G_t = \sum_{n=1}^{t} Q^{t-n} \Omega \Phi^{n-1}.$$

Let us substitute equation (49) into the objective function, given by formula (24), then

$$-\sum_{t=1}^{\infty}\beta^{t-1}\mathrm{E}_1\left[\lambda_c\hat{c}_t^2+\lambda_\pi(\pi_t-\vartheta\pi_{t-1})^2+\lambda_w\left(\pi_t+\widetilde{w}_t-\widetilde{w}_{t-1}-\vartheta_w\pi_{t-1}\right)^2\right].$$

Let us introduce some new notations:

$$\begin{aligned}K_t^\eta &= K_t-\eta K_{t-1},\\ G_t^\eta &= G_t-\eta G_{t-1},\end{aligned}$$

where $\eta=\delta,\ \vartheta,\ \vartheta_w,\ 1$. The row vectors of these matrices are denoted by $K_t^\eta(i:)$ and $G_t^\eta(i:)$, respectively. Using these notations the objective function takes the form

$$\begin{aligned}&- \sum_{t=1}^{\infty}\beta^{t-1}\left\{\lambda_c\left[K_t^\delta(2:)Y_0+G_t^\delta(2:)S_1\right]^2+\lambda_\pi\left[K_t^\vartheta(3:)Y_0+G_t^\vartheta(3:)S_1\right]^2\right\}\\ &- \lambda_w\sum_{t=1}^{\infty}\beta^{t-1}\left[\left(K_t^{\vartheta_w}(3:)+K_t^1(4:)\right)Y_0+\left(G_t^{\vartheta_w}(3:)+G_t^1(4:)\right)S_1\right]^2.\end{aligned}$$

The first-order condition is

$$-\frac{1}{2}\frac{\partial\left(\sum_{t=1}^{\infty}\beta^{t-1}\mathrm{E}_1\left[L_t\right]\right)}{\partial\left(d\tilde{e}_1\right)}=0,$$

i.e.

$$\begin{aligned}0 &= \lambda_c\sum_{t=1}^{\infty}\beta^{t-1}\left[K_t^\delta(2:)Y_0+G_t^\delta(2:)S_1\right]G_t^\delta(21)\\ &+ \lambda_\pi\sum_{t=1}^{\infty}\beta^{t-1}\left[K_t^\vartheta(3:)Y_0+G_t^\vartheta(3:)S_1\right]G_t^\vartheta(31)\\ &+ \lambda_w\sum_{t=1}^{\infty}\beta^{t-1}\left[K_t^{\vartheta_w}(3:)+K_t^1(4:)\right]Y_0\left[G_t^{\vartheta_w}(31)+G_t^1(41)\right]\\ &+ \lambda_w\sum_{t=1}^{\infty}\beta^{t-1}\left[G_t^{\vartheta_w}(3:)+G_t^1(4:)\right]S_1\left[G_t^{\vartheta_w}(31)+G_t^1(41)\right].\end{aligned}$$

The second-order condition is given by the coefficient of $d\tilde{e}_1$, which is evidently positive, since it is the square of an expression. Hence the objective function is concave, thus the first-order condition provides the maximum.

The first-order condition can be expressed alternatively as

$$0 = \sum_{j=1}^{6} \mathcal{K}_j^Y Y_0(j) + \sum_{s=1}^{9} \mathcal{K}_s^S S_1(s),$$

where

$$\begin{aligned} \mathcal{K}_j^Y &= \sum_{t=1}^{\infty} \beta^{t-1} \left[\lambda_c K_t^{\delta}(2j) G_t^{\delta}(21) + \lambda_\pi K_t^{\vartheta}(3j) G_t^{\vartheta}(31)\right] \\ &+ \sum_{t=1}^{\infty} \beta^{t-1} \lambda_w \left[K_t^{\vartheta_w}(3j) + K_t^1(4j)\right] \left[G_t^{\vartheta_w}(31) + G_t^1(41)\right], \end{aligned} \tag{50}$$

and

$$\begin{aligned} \mathcal{K}_s^S &= \sum_{t=1}^{\infty} \beta^{t-1} \left[\lambda_c G_t^{\delta}(2j) G_t^{\delta}(21) + \lambda_\pi G_t^{\vartheta}(3j) G_t^{\vartheta}(31)\right] \\ &+ \sum_{t=1}^{\infty} \beta^{t-1} \lambda_w \left[G_t^{\vartheta_w}(3j) + G_t^1(4j)\right] \left[G_t^{\vartheta_w}(31) + G_t^1(41)\right]. \end{aligned} \tag{51}$$

The above coefficients have closed form solutions. Since the eigenvalues of Q are different from each other, there exists a diagonal matrix M and an invertible matrix F, such that $M = FQ\hat{F}$, where $\hat{F} = F^{-1}$ and the diagonal elements of M are the eigenvalues of Q, which are denoted by μ_k. This implies that

$$K_t = FM^{t-1}\hat{F}, \quad G_t = \sum_{n=1}^{t} FM^{t-n}\hat{F}\Omega\Phi^{n-1}.$$

Using this one can show that the element in the ith row and jth column of K_t is given by

$$K_t(ij) = \sum_{k=1}^{6} K(ij,k)\mu_k^t, \tag{52}$$

where $K(ij,k) = f(ik)\hat{f}(kj)$, $f(ik)$ and $\hat{f}(kj)$ are the appropriate elements of F and $\hat{F}$, respectively. One can show by some calculations[12] that

$$G_t(is) = \sum_{k=1}^{6} \frac{\mu_k^t - \phi_s^t}{\mu_k - \phi_s} \widehat{\omega}(ik,s), \tag{53}$$

[12] For the calculations one has to use the fact that in this model $\mu_k \neq \phi_s$, for all k and s.

where ϕ_s is sth diagonal element of matrix Φ,

$$\widehat{\omega}(ik,s)=\sum_{l=1}^{6}\omega(ls)K(il,k),$$

and $\omega(ls)$ is the element in the lth row and sth column of matrix Ω.

Using equations (52) and (53) yields

$$\begin{aligned}K_t^\eta(ij) &= \sum_{k=1}^{6}K(ij,k)\kappa_\eta^k\mu_k^{t-1},\\ G_t^\eta(is) &= \sum_{k=1}^{6}\widehat{\omega}(ik,s)\frac{\kappa_\eta^k\mu_k^{t-1}-\psi_\eta^s\phi_s^{t-1}}{\mu_k-\phi_s},\end{aligned}$$

where $\kappa_\eta^k=\mu_k-\eta$, $\psi_\eta^s=\phi_s-\eta$. Substituting the above expressions into equations (50) and (51) and using the fact that $\phi_1=0$ one can obtain

$$\mathcal{K}_j^Y=\sum_{k=1}^{6}\sum_{l=1}^{6}\widehat{\mathcal{K}}_j^Y(kl),$$

where

$$\begin{aligned}\widehat{\mathcal{K}}_j^Y(kl) &= \lambda_c K(2j,k)\kappa_\delta^k\frac{\widehat{\omega}(2l,1)}{\mu_l}\left(\frac{\kappa_\delta^l}{1-\beta\mu_k\mu_l}+\delta\right)\\ &+ \lambda_\pi K(3j,k)\kappa_\vartheta^k\frac{\widehat{\omega}(3l,1)}{\mu_l}\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_k\mu_l}+\vartheta\right)\\ &+ \lambda_w\left(K(3j,k)\kappa_{\vartheta_w}^k+K(4j,k)\kappa_1^k\right)\frac{\widehat{\omega}(3l,1)}{\mu_l}\left(\frac{\kappa_{\vartheta_w}^l}{1-\beta\mu_k\mu_l}+\vartheta_w\right)\\ &+ \lambda_w\left(K(3j,k)\kappa_{\vartheta_w}^k+K(4j,k)\kappa_1^k\right)\frac{\widehat{\omega}(4l,1)}{\mu_l}\left(\frac{\kappa_1^l}{1-\beta\mu_k\mu_l}+1\right),\end{aligned}$$

and

$$\mathcal{K}_s^S=\sum_{k=1}^{6}\sum_{l=1}^{6}\widehat{\mathcal{K}}_s^S(kl),$$

where

$$
\begin{aligned}
&\widehat{\mathcal{K}}_s^S(kl) = \\
&\lambda_c \frac{\widehat{\omega}(2l,1)\widehat{\omega}(2k,s)}{(\mu_k-\phi_s)\mu_l}\left[\kappa_\delta^l\left(\frac{\kappa_\delta^l}{1-\beta\mu_k\mu_l}+\delta\right)-\psi_\delta^s\left(\frac{\kappa_\delta^l}{1-\beta\mu_l\phi_s}+\delta\right)\right] \\
&+\lambda_\pi \frac{\widehat{\omega}(3l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left[\kappa_\vartheta^l\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_k\mu_l}+\vartheta\right)-\psi_\vartheta^s\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_l\phi_s}+\vartheta\right)\right] \\
&+\lambda_w \frac{\widehat{\omega}(3l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left(\widehat{\omega}(3k,s)\kappa_{\vartheta_w}^k+\widehat{\omega}(4k,s)\kappa_1^k\right)\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_k\mu_l}+\vartheta_w\right) \\
&-\lambda_w \frac{\widehat{\omega}(3l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left(\widehat{\omega}(3k,s)\psi_{\vartheta_w}^s+\widehat{\omega}(4k,s)\psi_1^s\right)\left(\frac{\kappa_\vartheta^l}{1-\beta\phi_s\mu_l}+\vartheta_w\right) \\
&+\lambda_w \frac{\widehat{\omega}(4l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left(\widehat{\omega}(3k,s)\kappa_{\vartheta_w}^k+\widehat{\omega}(4k,s)\kappa_1^k\right)\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_k\mu_l}+1\right) \\
&-\lambda_w \frac{\widehat{\omega}(4l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left(\widehat{\omega}(3k,s)\psi_{\vartheta_w}^s+\widehat{\omega}(4k,s)\psi_1^s\right)\left(\frac{\kappa_\vartheta^l}{1-\beta\phi_s\mu_l}+1\right).
\end{aligned}
$$

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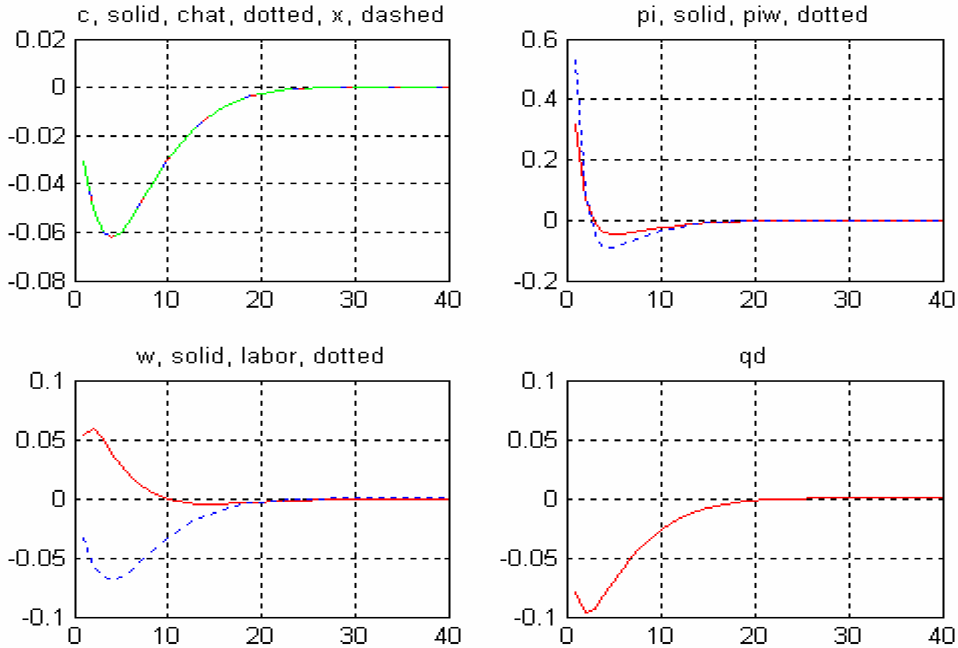
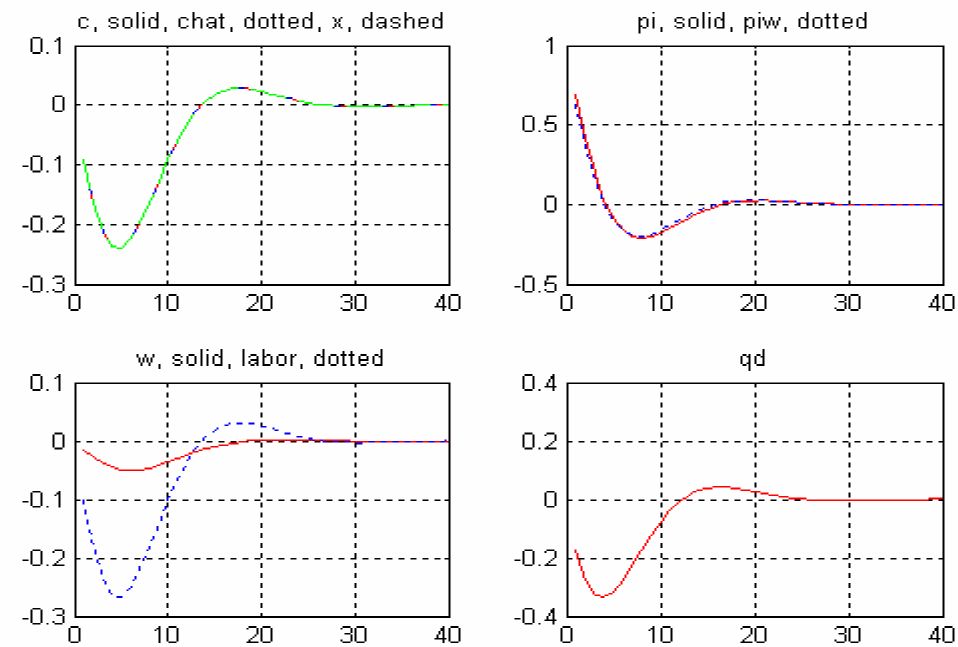
Figure 1

1 per cent initial deviation of the rate of inflation (π)
Small persistence of inflation



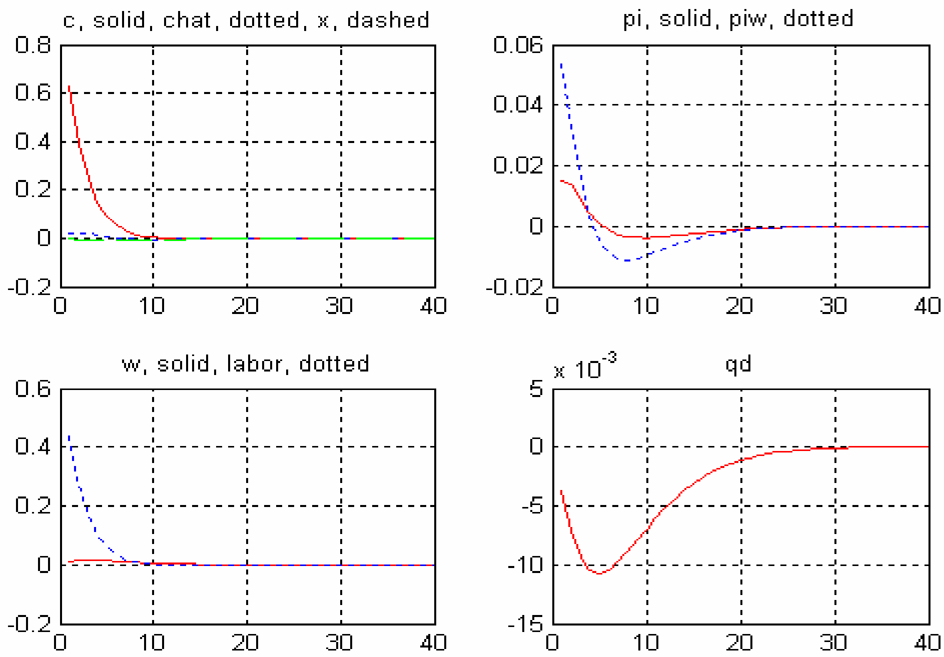
Large persistence of inflation



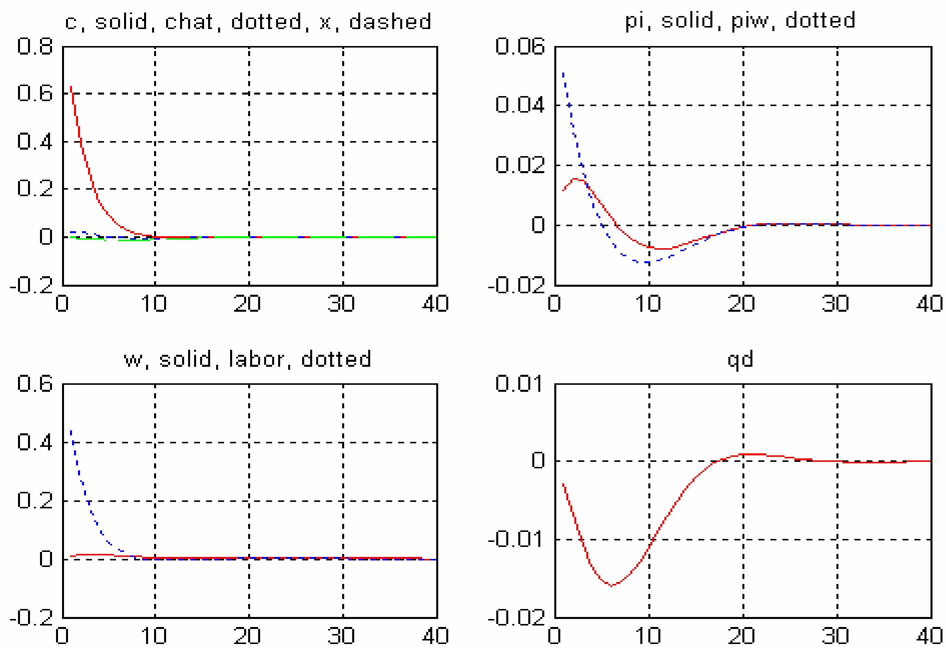
Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 2

1 per cent initial deviation of consumption (*c*)
Small persistence of inflation



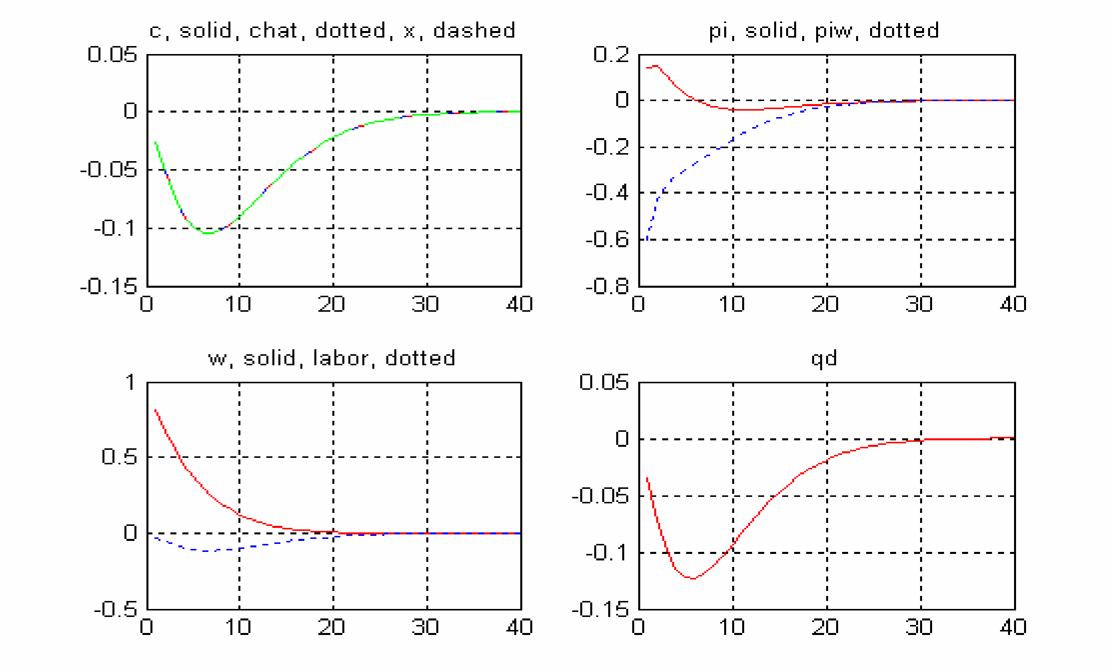
Large persistence of inflation



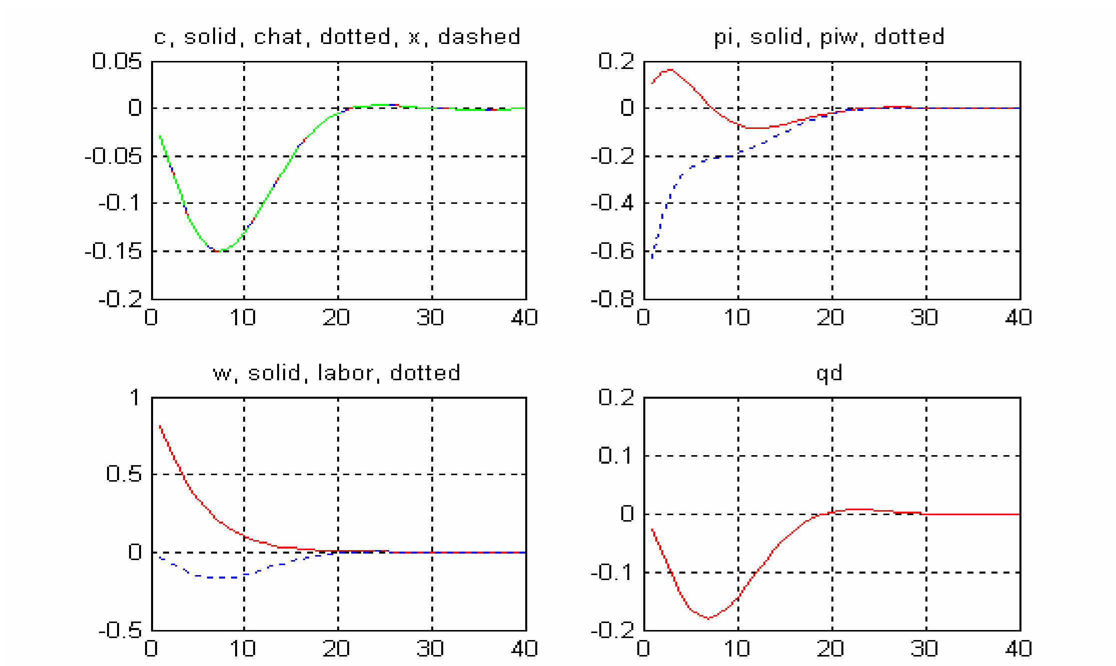
Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 3

1 per cent initial deviation of the real wage (w)
Small persistence of inflation



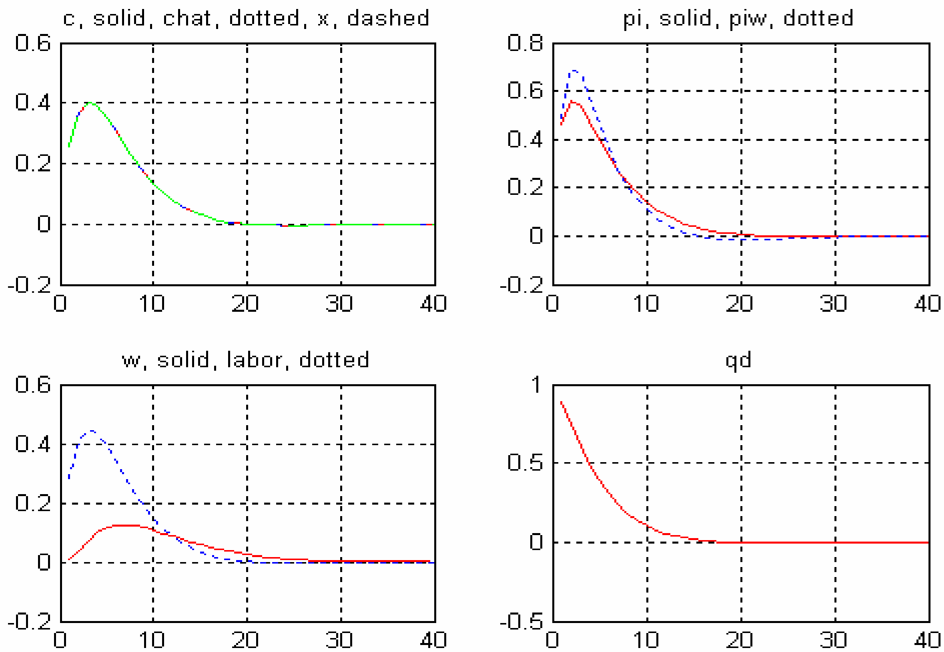
Large persistence of inflation



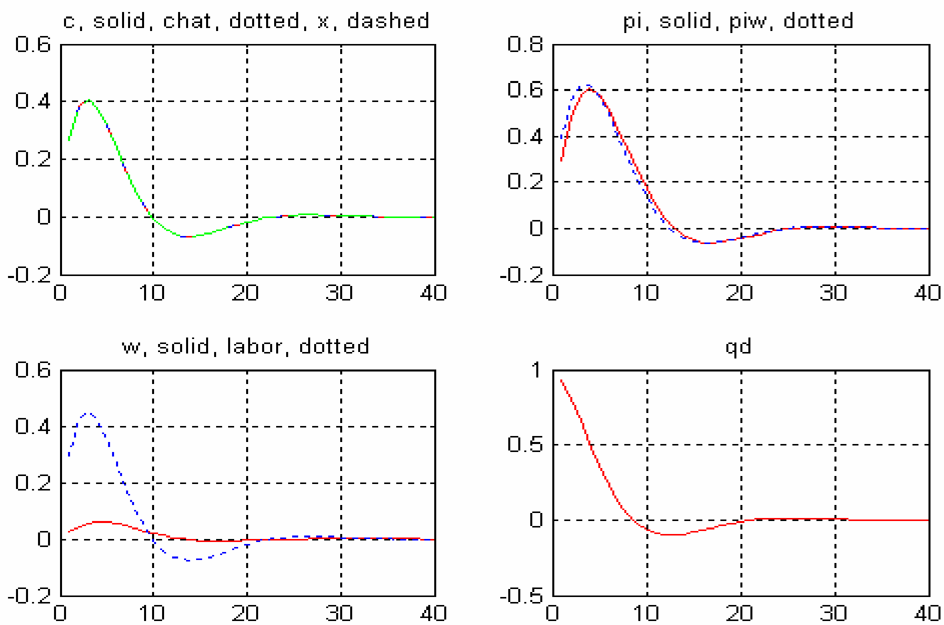
Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 4

1 per cent initial devaluation (*de*)
Small persistence of inflation



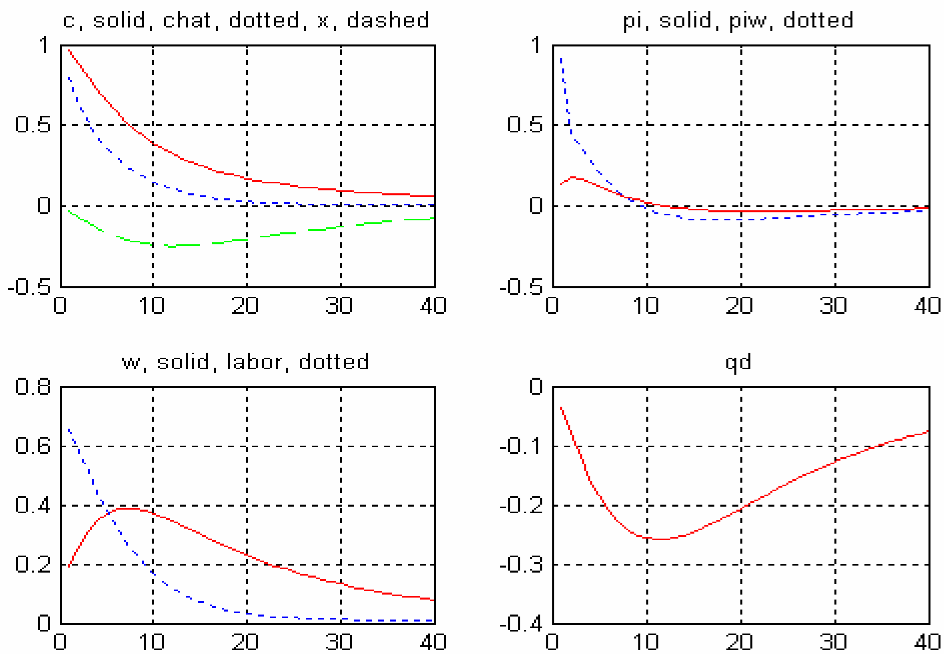
Large persistence of inflation



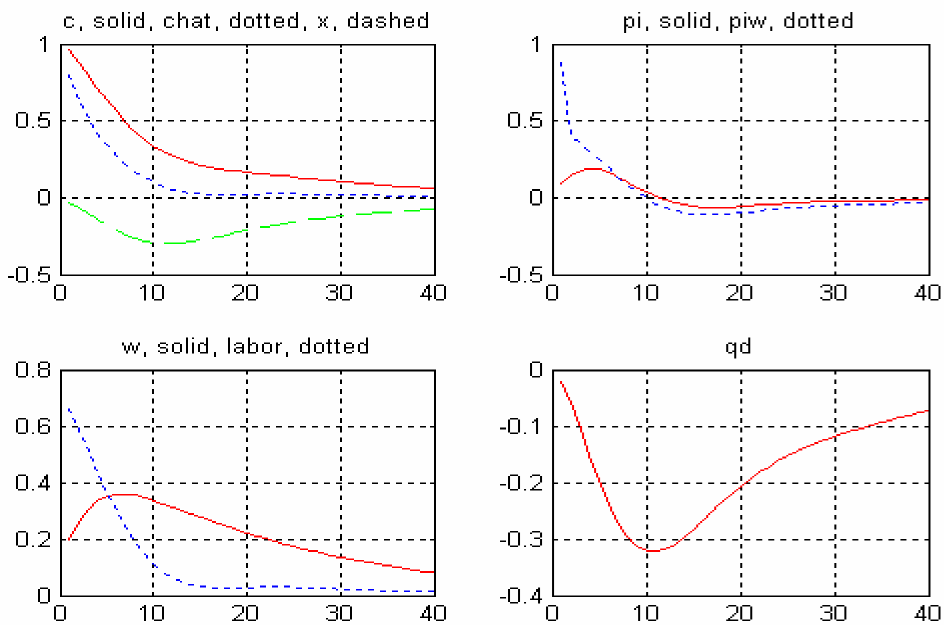
Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 5

1 per cent foreign-business-cycle (c^*) shock
Small persistence of inflation



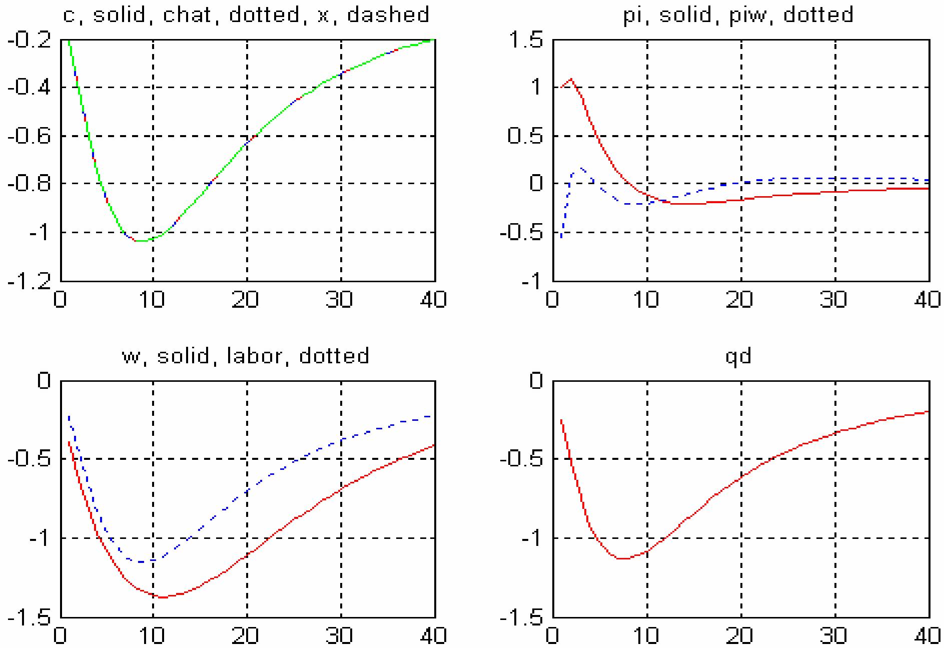
Large persistence of inflation



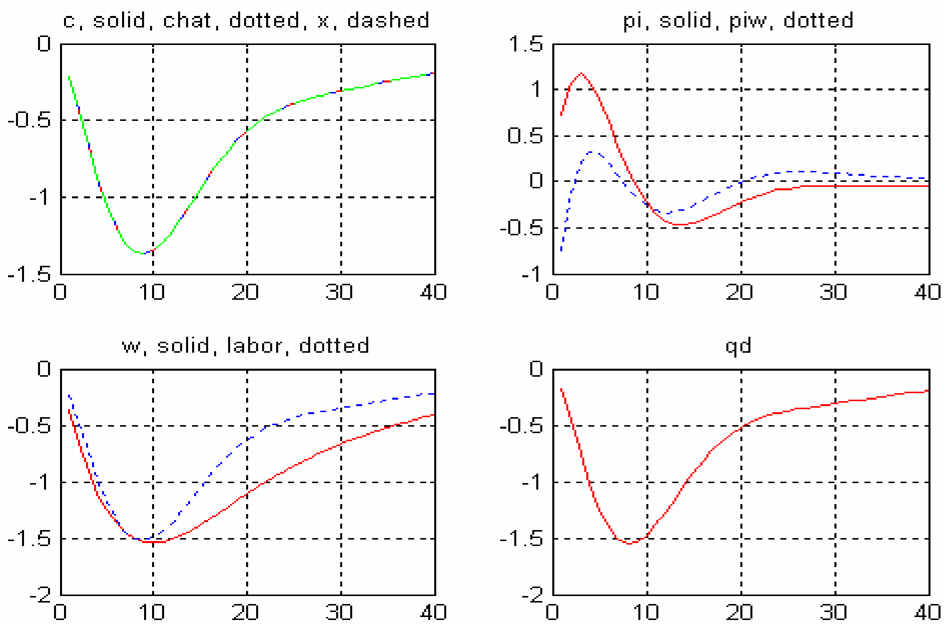
Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 6

0.08 per cent price (υ) shock
Small persistence of inflation



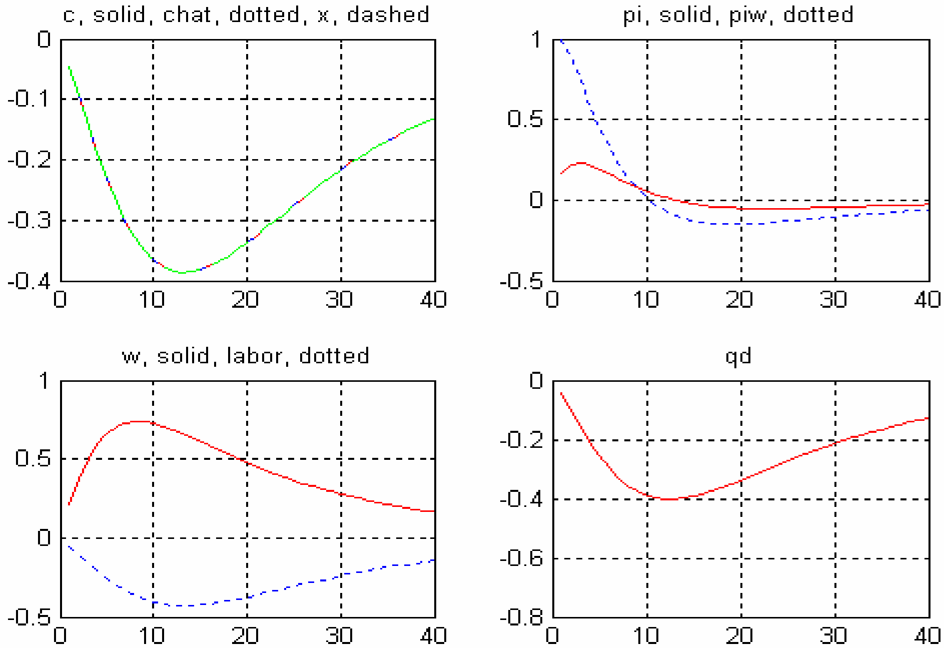
Large persistence of inflation



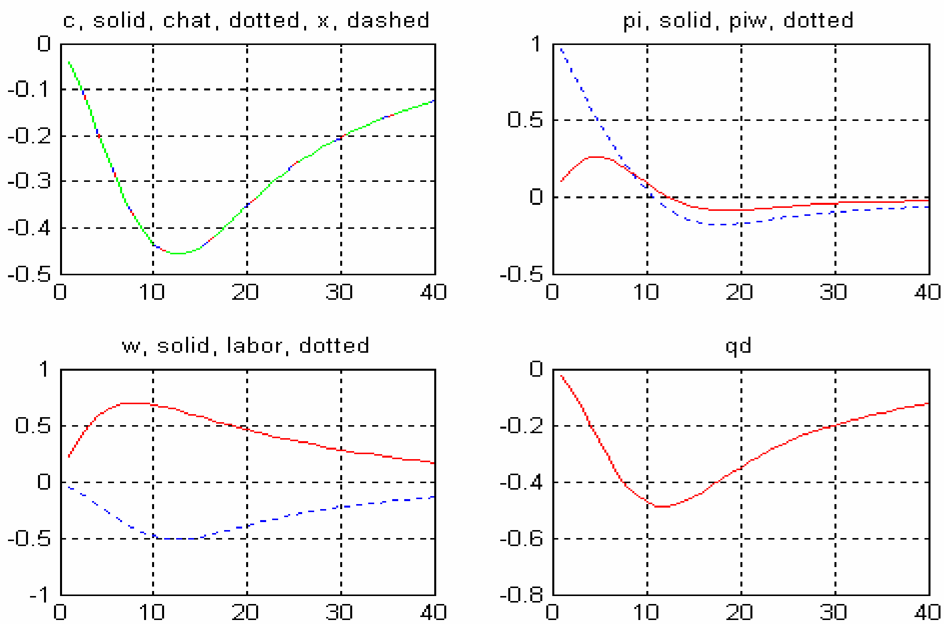
Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 7

0.06 per cent nominal-wage (υ^w) shock
Small persistence of inflation



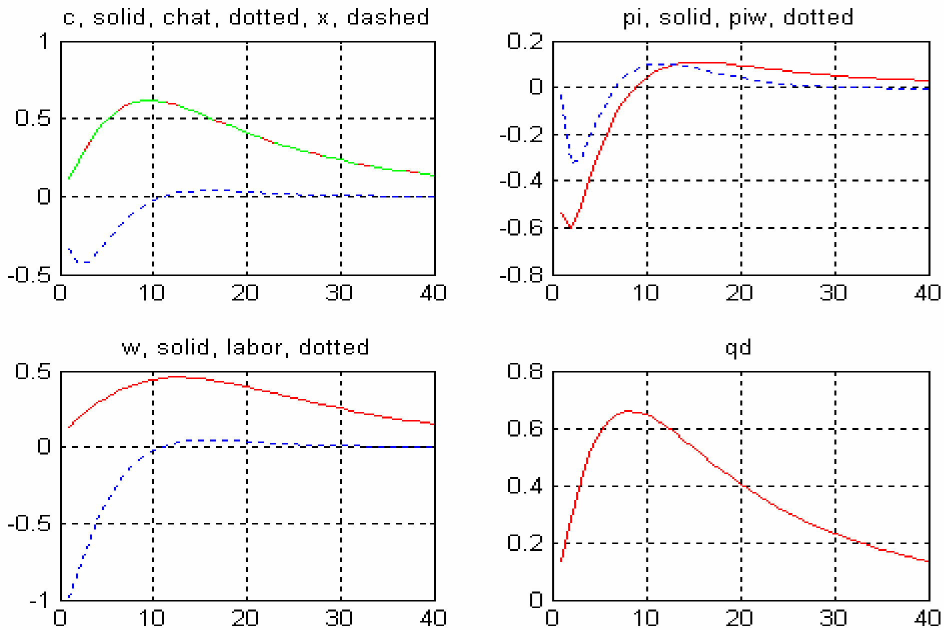
Large persistence of inflation



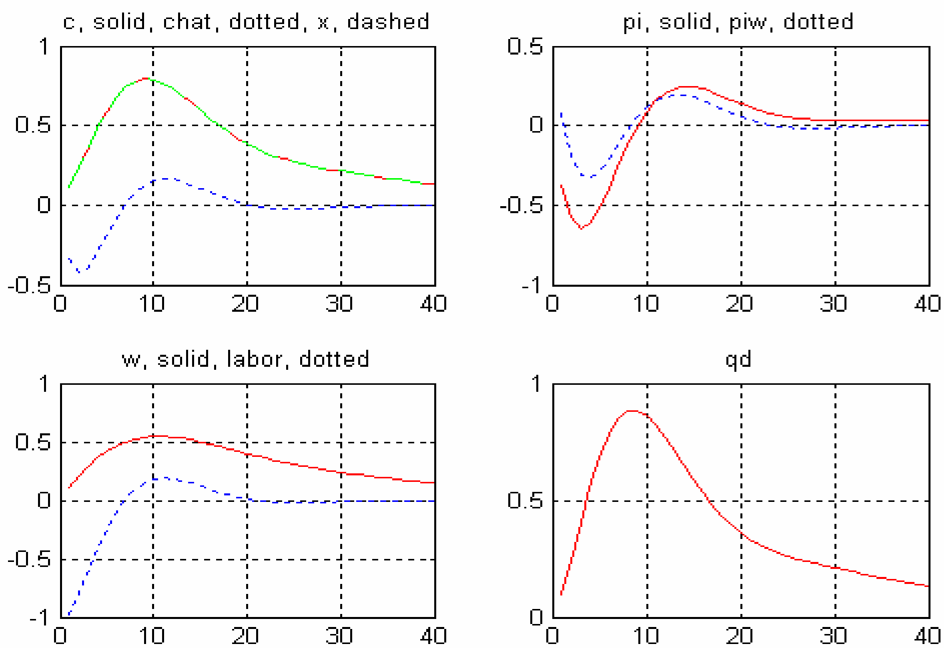
Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 8

1 per cent productivity *(A)* shock
Small persistence of inflation



Large persistence of inflation



Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.